AP* SOLUTIONS

Chapter 15    Learning from Categorical Data

Section 15.1 Exercise Set 1

15.1:  (a) With $df = k - 1 = 4 - 1 = 3$, the $P$-value is $P(X^2 \geq 6.4) = 0.094$. Because the $P$-value of 0.094 is greater than the significance level of $\alpha = 0.05$, we fail to reject $H_0$. We do not have convincing evidence that the sales are not equally divided among the four colors.

(b) With $df = k - 1 = 4 - 1 = 3$, the $P$-value is $P(X^2 \geq 15.3) = 0.002$. Because the $P$-value of 0.002 is less than the significance level of $\alpha = 0.01$, we reject $H_0$. We have convincing evidence that the sales are not equally divided among the four colors.

(c) With $df = k - 1 = 6 - 1 = 5$, the $P$-value is $P(X^2 \geq 13.7) = 0.018$. Because the $P$-value of 0.018 is less than the significance level of $\alpha = 0.05$, we reject $H_0$. We have convincing evidence that the sales are not equally divided among the six colors.

15.2:  (a) With 3 degrees of freedom, the approximate $P$-value is $P(X^2 \geq 6.62) = 0.085$.

(b) With 10 degrees of freedom, the approximate $P$-value is $P(X^2 \geq 16.97) = 0.075$.

(c) With 17 degrees of freedom, the approximate $P$-value is $P(X^2 \geq 30.19) = 0.025$.

15.3:  Using the five-step process (HMC$^3$):

Hypotheses (H):

We want to determine if there is convincing evidence that the proportions of Twitter users falling into each of the five categories are not all the same.

Define the following population characteristics of interest:

\[
\begin{align*}
p_1 & = \text{proportion of Twitter users in the IS (information sharing) category} \\
p_2 & = \text{proportion of Twitter users in the OC (opinions and complaints) category} \\
p_3 & = \text{proportion of Twitter users in the RT (random thoughts) category} \\
p_4 & = \text{proportion of Twitter users in the ME (me now / what am I doing now) category} \\
p_5 & = \text{proportion of Twitter users in the O (other) category}
\end{align*}
\]
State the appropriate hypotheses:

\[ H_0 : p_1 = p_2 = p_3 = p_4 = p_5 = 0.20 \]

\[ H_a : \text{At least one of the population proportions is not 0.20} \]

Method (M):

Because the answers to the four key questions are (Q) hypothesis testing, (S) sample data, (T) one categorical variable with more than two categories, and (N) one sample, a chi-square goodness-of-fit test will be considered.

The test statistic for this test is

\[ X^2 = \sum_{\text{all categories}} \frac{(\text{observed count} - \text{expected count})^2}{\text{expected count}} \]

When the null hypothesis is true, this statistic has approximately a chi-square distribution with \( k - 1 \) degrees of freedom.

A significance level of \( \alpha = 0.05 \) will be used for this test.

Check (C):

The 350 Twitter users in the sample were randomly selected. In addition, all expected counts in each category are all equal to \( 350(0.20) = 70 \), which is greater than 5, so the sample size is large enough. The two conditions necessary to use the chi-square goodness-of-fit test have been satisfied.

Calculations (C):

Test statistic:

\[
X^2 = \sum_{\text{all categories}} \frac{(\text{observed count} - \text{expected count})^2}{\text{expected count}}
\]

\[
= \frac{(51-70)^2}{70} + \frac{(61-70)^2}{70} + \frac{(64-70)^2}{70} + \frac{(101-70)^2}{70} + \frac{(73-70)^2}{70}
\]

\[
= 5.1571 + 1.1571 + 0.5143 + 13.7286 + 0.1286
\]

\[
= 20.686
\]

Degrees of freedom: \( df = k - 1 = 5 - 1 = 4 \)
P-value: The P-value is the area under the chi-square curve with 4 degrees of freedom and to the right of 20.686. Therefore, the P-value is equal to \( P(X^2 \geq 20.686) \approx 0 \).

Communicate Results (C):

Because the P-value of approximately 0 is less than the chosen significance level of \( \alpha = 0.05 \), we reject \( H_0 \). There is convincing evidence that the Twitter category proportions are not all equal.

15.4: (a) Using the five-step process (HMC\(^3\)):

Hypotheses (H):

We want to determine if there is convincing evidence that the proportion of home runs hit is not the same for all five directions.

Define the following population characteristics of interest:

- \( p_1 \) = proportion of home runs to left field
- \( p_2 \) = proportion of home runs to left center
- \( p_3 \) = proportion of home runs to center
- \( p_4 \) = proportion of home runs to right center
- \( p_5 \) = proportion of home runs to right field

State the appropriate hypotheses:

\[ H_0 : p_1 = p_2 = p_3 = p_4 = p_5 = 0.20 \]

\[ H_a : \text{At least one of the population proportions is not } 0.20 \]

Method (M):

Because the answers to the four key questions are (Q) hypothesis testing, (S) sample data, (T) one categorical variable with more than two categories, and (N) one sample, a chi-square goodness-of-fit test will be considered.

The test statistic for this test is

\[ X^2 = \sum_{\text{all categories}} \frac{(\text{observed count} - \text{expected count})^2}{\text{expected count}} \]
When the null hypothesis is true, this statistic has approximately a chi-square distribution with $k - 1$ degrees of freedom.

A significance level of $\alpha = 0.05$ will be used for this test.

Check (C):

We are told to assume that this sample of home runs is representative of home runs hit at Yankee Stadium. In addition, all expected counts in each category are all equal to $87(0.20) = 17.4$, which is greater than 5, so the sample size is large enough. The two conditions necessary to use the chi-square goodness-of-fit test have been satisfied.

Calculations (C):

Test statistic:

$$X^2 = \sum_{\text{all categories}} \frac{(\text{observed count} - \text{expected count})^2}{\text{expected count}}$$

$$= \frac{(18 - 17.4)^2}{17.4} + \frac{(10 - 17.4)^2}{17.4} + \frac{(7 - 17.4)^2}{17.4} + \frac{(18 - 17.4)^2}{17.4} + \frac{(34 - 17.4)^2}{17.4}$$

$$= 0.0207 + 3.1471 + 6.2161 + 0.0207 + 15.8368 = 25.2414$$

Degrees of freedom: $df = k - 1 = 5 - 1 = 4$

$P$-value: The $P$-value is the area under the chi-square curve with 4 degrees of freedom and to the right of 25.2414. Therefore, the $P$-value is equal to $P(X^2 \geq 25.2414) \approx 0$.

Communicate Results (C):

Because the $P$-value of approximately 0 is less than the chosen significance level of $\alpha = 0.05$, we reject $H_0$. There is convincing evidence that the proportions of home runs hit are not the same for all five directions.

(b) For home runs going to left center and center field the observed counts are significantly lower than the numbers that would have been expected if the proportion of home runs hit was the same for all five directions, while for right field the observed count is much high than the number that would have been expected.

15.5: Using the five-step process (HMC3):

Hypotheses (H):
We want to determine if there is convincing evidence to conclude that one or more of these three age groups buys a disproportionate share of lottery tickets.
Define the following population characteristics of interest:

\[ p_1 = \text{proportion of lottery ticket purchasers age 18-34} \]

\[ p_2 = \text{proportion of lottery ticket purchasers age 35-64} \]

\[ p_3 = \text{proportion of lottery ticket purchasers age 65 and over} \]

State the appropriate hypotheses:

\[ H_0 : p_1 = 0.35, p_2 = 0.51, p_3 = 0.14 \]

\[ H_a : \text{At least one of the population proportions is not as stated in } H_0 \]

Method (M):

Because the answers to the four key questions are (Q) hypothesis testing, (S) sample data, (T) one categorical variable with more than two categories, and (N) one sample, a chi-square goodness-of-fit test will be considered.

The test statistic for this test is

\[ X^2 = \sum_{\text{all categories}} \frac{(\text{observed count} - \text{expected count})^2}{\text{expected count}} \]

When the null hypothesis is true, this statistic has approximately a chi-square distribution with \( k - 1 \) degrees of freedom.

A significance level of \( \alpha = 0.05 \) will be used for this test.

Check (C):

We are told to suppose that these data resulted from a random sample of 200 lottery ticket purchasers. The expected counts in each category are 200(0.35) = 70 in the 18-34 age category, 200(0.51) = 102 in the 35-64 age category, and 200(0.14) = 28 in the 65 and over age category, which are all greater than 5, so the sample size is large enough. The two conditions necessary to use the chi-square goodness-of-fit test have been satisfied.
Calculations (C):

Test statistic:

\[ X^2 = \sum_{\text{all categories}} \frac{(\text{observed count} - \text{expected count})^2}{\text{expected count}} \]

\[ = \frac{(36 - 70)^2}{70} + \frac{(130 - 102)^2}{102} + \frac{(34 - 28)^2}{28} \]

\[ = 16.5143 + 7.6863 + 1.2857 \]

\[ = 25.4863 \]

Degrees of freedom: \( df = k - 1 = 3 - 1 = 2 \)

\( P \)-value: The \( P \)-value is the area under the chi-square curve with 2 degrees of freedom and to the right of 25.4863. Therefore, the \( P \)-value is equal to \( P(X^2 \geq 25.4863) \approx 0 \).

Communicate Results (C):

Because the \( P \)-value of approximately 0 is less than the chosen significance level of \( \alpha = 0.05 \), we reject \( H_0 \). There is convincing evidence that one or more of the age groups buys a disproportionate share of lottery tickets.

**Section 15.1 Exercise Set 2**

15.6:  
(a) With 6 degrees of freedom, the approximate \( P \)-value is \( P(X^2 \geq 14.44) = 0.025 \).

(b) With 9 degrees of freedom, the approximate \( P \)-value is \( P(X^2 \geq 16.91) = 0.050 \).

(c) With 20 degrees of freedom, the approximate \( P \)-value is \( P(X^2 \geq 32.32) = 0.040 \).

15.7:  
(a) With \( df = k - 1 = 4 - 1 = 3 \), the \( P \)-value is \( P(X^2 \geq 19.0) = 0.0003 \). Because the \( P \)-value of 0.0003 is less than the significance level of \( \alpha = 0.01 \), we reject \( H_0 \). We have convincing evidence that the percentages of nuts of the different types are not as advertised.

(b) If the sample consisted of only 40 nuts, the expected values for Type 1, Type 2, Type 3, and Type 4 nuts are 16, 12, 8, and 4, respectively. Because the expected value for Type 4 nuts is less than 5, the sample size is not large enough to use the chi-square goodness of fit test.
Hypotheses (H):

We want to determine if there is convincing evidence that the proportions of verbal aggression incidents are not the same for all five types of aggression.

Define the following population characteristics of interest:

\[ p_1 = \text{proportion of swearing attacks} \]
\[ p_2 = \text{proportion of competence attacks} \]
\[ p_3 = \text{proportion of character attacks} \]
\[ p_4 = \text{proportion of physical appearance attacks} \]
\[ p_5 = \text{proportion of other verbal aggression attacks} \]

State the appropriate hypotheses:

\[ H_0 : p_1 = p_2 = p_3 = p_4 = p_5 = 0.20 \]
\[ H_a : \text{At least one of the population proportions is not } 0.20 \]

Method (M):

Because the answers to the four key questions are (Q) hypothesis testing, (S) sample data, (T) one categorical variable with more than two categories, and (N) one sample, a chi-square goodness-of-fit test will be considered.

The test statistic for this test is

\[ X^2 = \sum_{\text{all categories}} \frac{\text{observed count} - \text{expected count}}{\text{expected count}}^2 \]

When the null hypothesis is true, this statistic has approximately a chi-square distribution with \( k - 1 \) degrees of freedom.

A significance level of \( \alpha = 0.01 \) will be used for this test.

Check (C):

We are told to assume that this sample of verbal aggression incidents is representative of all verbal aggression incidents in televised professional wrestling. In addition, all expected counts in each category are all equal to \( 804(0.20) = 160.8 \), which is greater
than 5, so the sample size is large enough. The two conditions necessary to use the 
chi-square goodness-of-fit test have been satisfied.

Calculations (C):

Test statistic:

\[
X^2 = \sum_{\text{all categories}} \left( \frac{\text{observed count} - \text{expected count}}{\text{expected count}} \right)^2 \] 

\[
= \frac{(219 - 160.8)^2}{160.8} + \frac{(166 - 160.8)^2}{160.8} + \frac{(127 - 160.8)^2}{160.8} + \frac{(75 - 160.8)^2}{160.8} + \frac{(217 - 160.8)^2}{160.8} 
\]

\[
= 21.0649 + 0.1682 + 7.1047 + 45.7813 + 19.6420 
\]

\[
= 93.7611 
\]

Degrees of freedom:  \( df = k - 1 = 5 - 1 = 4 \)

\( P \)-value: The \( P \)-value is the area under the chi-square curve with 4 degrees of freedom and to the right of 93.7611. Therefore, the \( P \)-value is equal to \( P(X^2 \geq 93.7611) \approx 0 \).

Communicate Results (C):

Because the \( P \)-value of approximately 0 is less than the chosen significance level of \( \alpha = 0.01 \), we reject \( H_0 \). There is convincing evidence that the proportions of verbal aggression incidents are not the same for all five types of aggression.

15.9: Using the five-step process (HMC\(^3\)):

Hypotheses (H):

We want to determine if there is convincing evidence that the proportions of male smoker lung cancer deaths are not the same for the four tar level categories.

Define the following population characteristics of interest:

\( p_1 = \) proportion of low tar level lung cancer deaths

\( p_2 = \) proportion of medium tar level lung cancer deaths

\( p_3 = \) proportion of high tar level lung cancer deaths

\( p_4 = \) proportion of very high tar level lung cancer deaths
State the appropriate hypotheses:

\[ H_0 : p_1 = p_2 = p_3 = p_4 = 0.25 \]

\[ H_a : \text{At least one of the population proportions is not } 0.25 \]

Method (M):

Because the answers to the four key questions are (Q) hypothesis testing, (S) sample data, (T) one categorical variable with more than two categories, and (N) one sample, a chi-square goodness-of-fit test will be considered.

The test statistic for this test is

\[ X^2 = \sum_{\text{all categories}} \frac{(\text{observed count} - \text{expected count})^2}{\text{expected count}} \]

When the null hypothesis is true, this statistic has approximately a chi-square distribution with \( k - 1 \) degrees of freedom.

A significance level of \( \alpha = 0.05 \) will be used for this test.

Check (C):

We are told to assume that this sample is representative of all male smokers who die of lung cancer. In addition, because the sample size was \( n = 1,194 \), all expected counts in each category are all equal to \( 1,194(0.25) = 298.5 \), which is greater than 5, so the sample size is large enough. The two conditions necessary to use the chi-square goodness-of-fit test have been satisfied.

Calculations (C):

Test statistic:

\[ X^2 = \sum_{\text{all categories}} \frac{(\text{observed count} - \text{expected count})^2}{\text{expected count}} \]

\[ = \frac{(103 - 298.5)^2}{298.5} + \frac{(378 - 298.5)^2}{298.5} + \frac{(563 - 298.5)^2}{298.5} + \frac{(150 - 298.5)^2}{298.5} \]

\[ = 128.041 + 21.173 + 234.373 + 73.877 \]

\[ = 457.464 \]

Degrees of freedom: \( df = k - 1 = 4 - 1 = 3 \)
P-value: The P-value is the area under the chi-square curve with 3 degrees of freedom and to the right of 457.464. Therefore, the P-value is equal to \( P(X^2 \geq 457.464) \approx 0 \).

Communicate Results (C):

Because the P-value of approximately 0 is less than the chosen significance level of \( \alpha = 0.05 \), we reject \( H_0 \). There is convincing evidence that the proportions of male smoker lung cancer deaths are not the same for the four tar level categories.

15.10: (a) Using the five-step process (HMC^3):

Hypotheses (H):

We want to determine if there is convincing evidence that the proportions of the age at which male low-tar cigarette smokers began smoking are not as specified.

Define the following population characteristics of interest:

\[ p_1 = \text{proportion of male low-tar cigarette smokers who started smoking before age 16} \]

\[ p_2 = \text{proportion of male low-tar cigarette smokers who started smoking at ages 16 or 17} \]

\[ p_3 = \text{proportion of male low-tar cigarette smokers who started smoking at ages 18, 19, or 20} \]

\[ p_4 = \text{proportion of male low-tar cigarette smokers who started smoking at age 21 or older} \]

State the appropriate hypotheses:

\[ H_0 : p_1 = 0.25, p_2 = 0.20, p_3 = 0.30, p_4 = 0.25 \]

\[ H_a : \text{At least one of the population proportions is not as stated in } H_0 \]

Method (M):

Because the answers to the four key questions are (Q) hypothesis testing, (S) sample data, (T) one categorical variable with more than two categories, and (N) one sample, a chi-square goodness-of-fit test will be considered.
The test statistic for this test is

\[ X^2 = \sum \frac{(\text{observed count} - \text{expected count})^2}{\text{expected count}} \]

When the null hypothesis is true, this statistic has approximately a chi-square distribution with \( k - 1 \) degrees of freedom.

A significance level of \( \alpha = 0.05 \) will be used for this test.

Check (C):

We are told (from exercise 15.9) to suppose that these data resulted from a representative sample. The expected counts in each category are 1,031(0.25) = 257.75 for the <16 age category, 1,031(0.20) = 206.2 in the 16-17 age category, 1,031(0.30) = 309.3 in the 18-20 age category, and 1,031(0.25) = 257.75 in the 21 and over age category, which are all greater than 5, so the sample size is large enough. The two conditions necessary to use the chi-square goodness-of-fit test have been satisfied.

Calculations (C):

Test statistic:

\[ X^2 = \sum \frac{(\text{observed count} - \text{expected count})^2}{\text{expected count}} \]

\[ \begin{align*}
\sum &= \frac{(237 - 257.75)^2}{257.75} + \frac{(258 - 206.2)^2}{206.2} + \frac{(320 - 309.3)^2}{309.3} + \frac{(216 - 257.75)^2}{257.75} \\
&= 1.6705 + 13.0128 + 0.3702 + 6.7626 \\
&= 21.8161
\end{align*} \]

Degrees of freedom: \( df = k - 1 = 4 - 1 = 3 \)

\( P \)-value: The \( P \)-value is the area under the chi-square curve with 3 degrees of freedom and to the right of 21.8161. Therefore, the \( P \)-value is equal to \( P(X^2 \geq 21.8161) \approx 0 \).

Communicate Results (C):

Because the \( P \)-value of approximately 0 is less than the chosen significance level of \( \alpha = 0.05 \), we reject \( H_0 \). There is convincing evidence that the proportions of the age at which male low-tar cigarette smokers began smoking are not as specified.
(b) The 16-17 age category consists of two years, and the 18-20 age category consists of three years, so it makes sense that the proportion \( p = 0.5 \) for ages 16-20 should be distributed as 0.2 and 0.3 in the two age categories.

**Section 15.1 Additional Exercises**

15.11: (a) With 13 degrees of freedom, the approximate \( P \)-value is \( P(X^2 \geq 34.52) = 0.001 \).

(b) With 16 degrees of freedom, the approximate \( P \)-value is \( P(X^2 \geq 39.25) = 0.001 \).

(c) With 19 degrees of freedom, the approximate \( P \)-value is \( P(X^2 \geq 26.00) = 0.130 \).

15.12: The null hypothesis would be rejected for the \( X^2 \) and df pairs in parts (a) and (b).

15.13: Because the \( P \)-value of less than 0.001 is smaller than any reasonable significance level, we would reject \( H_0 \). There is convincing evidence that the response proportions are not each 0.5.

15.14: (a) Using the five-step process (HMC³):  

**Hypotheses (H):**

We want to determine if there is convincing evidence to conclude that the researcher’s theory is not correct.

Define the following population characteristics of interest:

\[
p_1 = \text{proportion of all plants that have phenotype 1}
\]

\[
p_2 = \text{proportion of all plants that have phenotype 2}
\]

\[
p_3 = \text{proportion of all plants that have phenotype 3}
\]

State the appropriate hypotheses:

\[
H_0 : p_1 = 0.25, \ p_2 = 0.50, \ p_3 = 0.25
\]

\[
H_a : \text{At least one of the population proportions is not as stated in } H_0
\]

**Method (M):**

Because the answers to the four key questions are (Q) hypothesis testing, (S) sample data, (T) one categorical variable with more than two categories, and (N) one sample, a chi-square goodness-of-fit test will be considered.
The test statistic for this test is

\[ X^2 = \sum_{\text{all categories}} \frac{(\text{observed count} - \text{expected count})^2}{\text{expected count}} \]

When the null hypothesis is true, this statistic has approximately a chi-square distribution with \( k - 1 \) degrees of freedom.

A significance level of \( \alpha = 0.05 \) will be used for this test.

Check (C):

We are told these data resulted from a random sample of 200 plants. The expected counts in each category are 200(0.25) = 50 for phenotype 1, 200(0.50) = 100 for phenotype 2, and 200(0.25) = 50 for phenotype 3, which are all greater than 5, so the sample size is large enough. The two conditions necessary to use the chi-square goodness-of-fit test have been satisfied.

Calculations (C):

Test statistic:

\[ X^2 = \sum_{\text{all categories}} \frac{(\text{observed count} - \text{expected count})^2}{\text{expected count}} = 4.63 \]

Degrees of freedom: \( \text{df} = k - 1 = 3 - 1 = 2 \)

\( P \)-value: The \( P \)-value is the area under the chi-square curve with 2 degrees of freedom and to the right of 4.63. Therefore, the \( P \)-value is equal to \( P(X^2 \geq 4.63) = 0.099 \).

Communicate Results (C):

Because the \( P \)-value of 0.099 is greater than the significance level of \( \alpha = 0.05 \), we fail to reject \( H_0 \). There is not convincing evidence that the researcher’s theory is incorrect.

(b) The analysis and conclusion would not differ from those in part (a).

15.15: Using the five-step process (HMC): 3

Hypotheses (H):

We want to determine if there is convincing evidence to conclude that the data from this experiment are not consistent with Mendel’s laws.
Define the following population characteristics of interest:

\[ p_1 = \text{proportion of tall cut-leaf phenotypes} \]
\[ p_2 = \text{proportion of tall potato-leaf phenotypes} \]
\[ p_3 = \text{proportion of dwarf-cut leaf phenotypes} \]
\[ p_4 = \text{proportion of dwarf potato-leaf phenotypes} \]

State the appropriate hypotheses:

\[ H_0 : p_1 = \frac{9}{16}, p_2 = \frac{3}{16}, p_3 = \frac{3}{16}, p_4 = \frac{1}{16} \]

\[ H_a : \text{At least one of the population proportions is not as stated in } H_0 \]

Method (M):

Because the answers to the four key questions are (Q) hypothesis testing, (S) sample data, (T) one categorical variable with more than two categories, and (N) one sample, a chi-square goodness-of-fit test will be considered.

The test statistic for this test is

\[ X^2 = \sum_{\text{all categories}} \frac{(\text{observed count} - \text{expected count})^2}{\text{expected count}} \]

When the null hypothesis is true, this statistic has approximately a chi-square distribution with \( k - 1 \) degrees of freedom.

A significance level of \( \alpha = 0.01 \) will be used for this test.

Check (C):

We are not told how the sample was selected or if it is representative of all such tomato plants, so must assume that one of these conditions is satisfied. The expected counts in each category are \( 1,611 \left( \frac{9}{16} \right) = 906.188 \) in the tall cut-leaf phenotype category, \( 1,611 \left( \frac{3}{16} \right) = 302.063 \) in both the tall potato-leaf and dwarf cut-leaf phenotype categories, and \( 1,611 \left( \frac{1}{16} \right) = 100.688 \) in the dwarf potato-leaf phenotype category.
category, which are all greater than 5, so the sample size is large enough. The two conditions necessary to use the chi-square goodness-of-fit test have been satisfied.

Calculations (C):

Test statistic:

$$X^2 = \sum \frac{(\text{observed count} - \text{expected count})^2}{\text{expected count}}$$

$$= \frac{(926 - 906.188)^2}{906.188} + \frac{(288 - 302.063)^2}{302.063} + \frac{(293 - 302.063)^2}{302.063} + \frac{(104 - 100.688)^2}{100.688}$$

$$= 0.433172 + 0.654679 + 0.271894 + 0.108977$$

$$= 1.46872$$

Degrees of freedom: $\text{df} = k - 1 = 4 - 1 = 3$

$P$-value: The $P$-value is the area under the chi-square curve with 3 degrees of freedom and to the right of 1.46872. Therefore, the $P$-value is equal to $P(X^2 \geq 1.46872) = 0.690$.

Communicate Results (C):

Because the $P$-value of 0.690 is much greater than the chosen significance level of $\alpha = 0.01$, we fail to reject $H_0$. There is not convincing evidence to conclude that the data from this experiment are not consistent with Mendel’s laws.

15.16: Using the five-step process (HMC³):

Hypotheses (H):

We want to determine if there is convincing evidence that the proportion of fatal accidents is not the same for all days of the week.

Define the following population characteristics of interest:

$$p_1 = \text{proportion of fatal accidents on Sunday}$$
$$p_2 = \text{proportion of fatal accidents on Monday}$$
$$p_3 = \text{proportion of fatal accidents on Tuesday}$$
$$p_4 = \text{proportion of fatal accidents on Wednesday}$$
$$p_5 = \text{proportion of fatal accidents on Thursday}$$
$p_6 =$ proportion of fatal accidents on Friday  
$p_7 =$ proportion of fatal accidents on Saturday

State the appropriate hypotheses:

$H_0 : p_1 = p_2 = p_3 = p_4 = p_5 = p_6 = p_7 = \frac{1}{7}$

$H_a : $ At least one of the population proportions is not $\frac{1}{7}$

Method (M):

Because the answers to the four key questions are (Q) hypothesis testing, (S) sample data, (T) one categorical variable with more than two categories, and (N) one sample, a chi-square goodness-of-fit test will be considered.

The test statistic for this test is

$$X^2 = \sum_{\text{all categories}} \frac{(\text{observed count} - \text{expected count})^2}{\text{expected count}}$$

When the null hypothesis is true, this statistic has approximately a chi-square distribution with $k - 1$ degrees of freedom.

A significance level of $\alpha = 0.05$ will be used for this test.

Check (C):

We are told that the data came from a random sample. In addition, all expected counts in each category are all equal to $100\left(\frac{1}{7}\right) = 14.2857$, which is greater than 5, so the sample size is large enough. The two conditions necessary to use the chi-square goodness-of-fit test have been satisfied.

Calculations (C):

Test statistic:
\[ X^2 = \sum_{\text{all categories}} \frac{(\text{observed count} - \text{expected count})^2}{\text{expected count}} \]

\[ = \frac{(14 - 14.2857)^2}{14.2857} + \frac{(13 - 14.2857)^2}{14.2857} + \cdots + \frac{(15 - 14.2857)^2}{14.2857} \]

\[ = 0.005714 + 0.115714 + \cdots + 0.035714 = 1.08 \]

Degrees of freedom:  \( df = k - 1 = 7 - 1 = 6 \)

\( P \)-value: The \( P \)-value is the area under the chi-square curve with 6 degrees of freedom and to the right of 1.08. Therefore, the \( P \)-value is equal to \( P(X^2 \geq 1.08) = 0.982 \).

Communicate Results (C):

Because the \( P \)-value of 0.982 is greater than the significance level of \( \alpha = 0.05 \), we fail to reject \( H_0 \). There is not convincing evidence that the proportion of fatal accidents is not the same for all days of the week.

15.17: Using the five-step process (HMC^3):

Hypotheses (H):

We want to determine if there is convincing evidence that the 1-day old bobwhites have a color preference.

Define the following population characteristics of interest:

\[ p_1 = \text{proportion of bobwhites whose first peck is blue} \]
\[ p_2 = \text{proportion of bobwhites whose first peck is green} \]
\[ p_3 = \text{proportion of bobwhites whose first peck is yellow} \]
\[ p_4 = \text{proportion of bobwhites whose first peck is red} \]

State the appropriate hypotheses:

\[ H_0 : p_1 = p_2 = p_3 = p_4 = 0.25 \]
\[ H_a : \text{At least one of the population proportions is not 0.25} \]

Method (M):
Because the answers to the four key questions are (Q) hypothesis testing, (S) sample data, (T) one categorical variable with more than two categories, and (N) one sample, a chi-square goodness-of-fit test will be considered.

The test statistic for this test is

\[ X^2 = \sum_{\text{all categories}} \frac{(\text{observed count} - \text{expected count})^2}{\text{expected count}} \]

When the null hypothesis is true, this statistic has approximately a chi-square distribution with \( k - 1 \) degrees of freedom.

A significance level of \( \alpha = 0.01 \) will be used for this test.

Check (C):

We are not told how the 1-day old bobwhites in the study were selected, so we must assume they were randomly selected or that they form a representative sample. In addition, all expected counts in each category are all equal to \( 33(0.25) = 8.25 \), which is greater than 5, so the sample size is large enough. The two conditions necessary to use the chi-square goodness-of-fit test have been satisfied.

Calculations (C):

Test statistic:

\[ X^2 = \sum_{\text{all categories}} \frac{(\text{observed count} - \text{expected count})^2}{\text{expected count}} \]

\[ = \frac{(16 - 8.25)^2}{8.25} + \frac{(8 - 8.25)^2}{8.25} + \frac{(6 - 8.25)^2}{8.25} + \frac{(3 - 8.25)^2}{8.25} \]

\[ = 7.28030 + 0.00758 + 0.61364 + 3.34091 \]

\[ = 11.2424 \]

Degrees of freedom: \( df = k - 1 = 4 - 1 = 3 \)

\( P \)-value: The \( P \)-value is the area under the chi-square curve with 3 degrees of freedom and to the right of 11.2424. Therefore, the \( P \)-value is equal to \( P(X^2 \geq 11.2424) = 0.0105 \).

Communicate Results (C):
Because the $P$-value of 0.0105 is greater than the chosen significance level of $\alpha = 0.01$, we fail to reject $H_0$. There is not convincing evidence that there is a color preference.

15.18: Using the five-step process (HMC$^3$):

**Hypotheses (H):**

We want to determine if there is convincing evidence that the proportions of homicides are not the same for the four seasons.
Define the following population characteristics of interest:

\[ p_1 = \text{proportion of homicides in Winter} \]
\[ p_2 = \text{proportion of homicides in Spring} \]
\[ p_3 = \text{proportion of homicides in Summer} \]
\[ p_4 = \text{proportion of homicides in Fall} \]

State the appropriate hypotheses:

\[ H_0 : p_1 = p_2 = p_3 = p_4 = 0.25 \]
\[ H_a : \text{At least one of the population proportions is not } 0.25 \]

Method (M):

Because the answers to the four key questions are (Q) hypothesis testing, (S) sample data, (T) one categorical variable with more than two categories, and (N) one sample, a chi-square goodness-of-fit test will be considered.

The test statistic for this test is

\[ X^2 = \sum_{\text{all categories}} \frac{(\text{observed count} - \text{expected count})^2}{\text{expected count}} \]

When the null hypothesis is true, this statistic has approximately a chi-square distribution with \( k - 1 \) degrees of freedom.

A significance level of \( \alpha = 0.05 \) will be used for this test.

Check (C):

We are not told that this sample is representative of all homicides, or that the sample of homicides was randomly selected, so we must proceed with caution. In addition, because the sample size was \( n = 1,361 \), all expected counts in each category are all equal to \( 1,361(0.25) = 340.25 \), which is greater than 5, so the sample size is large enough. Assuming the sample was selected appropriately, the two conditions necessary to use the chi-square goodness-of-fit test have been satisfied.
Calculations (C):

Test statistic:

\[ X^2 = \sum_{\text{all categories}} \left( \frac{\text{observed count} - \text{expected count}}{\text{expected count}} \right)^2 \]

\[ = \left( \frac{328 - 340.25}{340.25} \right)^2 + \left( \frac{334 - 340.25}{340.25} \right)^2 + \left( \frac{372 - 340.25}{340.25} \right)^2 + \left( \frac{327 - 340.25}{340.25} \right)^2 \]

\[ = 0.44104 + 0.11481 + 2.96271 + 0.51598 \]

\[ = 4.03454 \]

Degrees of freedom: \( df = k - 1 = 4 - 1 = 3 \)

\( P \)-value: The \( P \)-value is the area under the chi-square curve with 3 degrees of freedom and to the right of 4.03454. Therefore, the \( P \)-value is equal to \( P(X^2 \geq 4.03454) = 0.258 \).

Communicate Results (C):

Because the \( P \)-value of 0.258 is greater than the significance level of \( \alpha = 0.05 \), we fail to reject \( H_0 \). There is not convincing evidence that the proportions of homicides are not the same for the four seasons. The data do not support the theory that the homicide rate is not the same for the four seasons.

Section 15.2 Exercise Set 1

15.19: Using the five-step process (HMC^3):

Hypotheses (H):

We want to determine if there is convincing evidence that the proportion falling in the three credit card response categories is not the same for all three years.

The hypotheses of interest are:

\( H_0: \) The proportions falling into the three credit card response categories are the same for all three years

\( H_a: \) The proportions falling into the three credit card response categories are not the same for all three years
Method (M):

The purpose of this study is to compare three populations (the three years 2004, 2005, and 2006) on the basis of one categorical variable (credit card payment response), so a chi-square test of homogeneity will be considered.

The test statistic for this test is

$$X^2 = \sum_{\text{all categories}} \left( \frac{\text{observed count} - \text{expected count}}{\text{expected count}} \right)^2$$

When the null hypothesis is true, this statistic has approximately a chi-square distribution with (number of rows – 1)(number of columns – 1) degrees of freedom.

A significance level of $\alpha = 0.05$ will be used for this test.

Check (C):

We are told that in each year (2004, 2005, and 2006) a random sample of people age 18 or older was selected. The table below shows observed counts and, in parentheses, expected counts. Note that all expected counts are at least 5.

<table>
<thead>
<tr>
<th></th>
<th>2004</th>
<th>2005</th>
<th>2006</th>
</tr>
</thead>
<tbody>
<tr>
<td>Definitely Will/Probably Will</td>
<td>40 (43.33)</td>
<td>50 (43.33)</td>
<td>40 (43.33)</td>
</tr>
<tr>
<td>Might/Might Not/Probably Not</td>
<td>180 (176.67)</td>
<td>190 (176.67)</td>
<td>160 (176.67)</td>
</tr>
<tr>
<td>Definitely Will Not</td>
<td>781 (781)</td>
<td>761 (781)</td>
<td>801 (781)</td>
</tr>
</tbody>
</table>

Because the samples were independent random samples and all expected counts are at least 5, the chi-square test of homogeneity is appropriate.

Calculations (C):

Test statistic:

$$X^2 = \sum_{\text{all categories}} \left( \frac{\text{observed count} - \text{expected count}}{\text{expected count}} \right)^2$$

$$= \frac{(40 - 43.3333)^2}{43.3333} + \frac{(50 - 43.3333)^2}{43.3333} + \cdots + \frac{(761 - 781)^2}{781} + \frac{(801 - 781)^2}{781}$$

$$= 0.256 + 1.026 + \cdots + 0.512 + 0.512$$

$$= 5.204$$

Degrees of freedom: $df = (\text{number of rows} - 1)(\text{number of columns} - 1) = (2)(2) = 4$
P-value: The $P$-value is the area under the chi-square curve with 4 degrees of freedom and to the right of 5.204. Therefore, the $P$-value is equal to $P(X^2 \geq 5.204) = 0.267$.

Communicate Results (C):

Because the $P$-value of 0.267 is greater than the specified significance level of $\alpha = 0.05$, we fail to reject $H_0$. There is not convincing evidence that the proportions falling into the three credit response categories are not all the same for all three years.

15.20: (a) Using the five-step process (HMC$^3$):

Hypotheses (H):

We want to determine if there is convincing evidence that the proportions in the two donation categories are not the same for all three types of requests.

The hypotheses of interest are:

$H_0$: donation proportions are the same for all three gift types

$H_a$: donation proportions are not the same for all three gift types

Method (M):

The purpose of this study is to compare three treatments (the three gift categories) on the basis of one categorical variable (donation result), so a chi-square test of homogeneity will be considered.

The test statistic for this test is

$$X^2 = \sum_{\text{all categories}} \frac{(\text{observed count} - \text{expected count})^2}{\text{expected count}}$$

When the null hypothesis is true, this statistic has approximately a chi-square distribution with $(\text{number of rows} - 1)(\text{number of columns} - 1)$ degrees of freedom.

A significance level of $\alpha = 0.01$ will be used for this test.

Check (C):

We are told that potential donors who were to receive the letter were assigned, at random, to one of the three groups (no gift, small gift, large gift). The table below shows observed counts and, in parentheses, expected counts. Note that all expected counts are at least 5.
Because the treatments were randomly assigned and all expected counts are at least 5, the chi-square test of homogeneity is appropriate.

Calculations (C):

Test statistic:

\[
X^2 = \sum_{\text{all categories}} \frac{(\text{observed count} - \text{expected count})^2}{\text{expected count}}
\]

\[
= \frac{(397 - 514.51)^2}{514.51} + \frac{(2,865 - 2,747.49)^2}{2,747.49} + \cdots + \frac{(691 - 527.92)^2}{527.92} + \frac{(2,656 - 2,819.08)^2}{2,819.08}
\]

\[
= 26.839 + 5.026 + \cdots + 50.378 + 9.434 = 96.506
\]

Degrees of freedom: \( df = (\text{number of rows} - 1)(\text{number of columns} - 1) = (2)(1) = 2 \)

\( P \)-value: The \( P \)-value is the area under the chi-square curve with 2 degrees of freedom and to the right of 96.506. Therefore, the \( P \)-value is equal to \( P(X^2 \geq 96.506) \approx 0 \).

Communicate Results (C):

Because the \( P \)-value of approximately 0 is smaller than the specified significance level of \( \alpha = 0.01 \), we reject \( H_0 \). There is convincing evidence that the donation proportions are not the same for all three gift types.

(b) The result of part (a) tells us that the level of the gift seems to make a difference. Looking at the data given, 12% of those receiving no gift made a donation, 14% of those receiving a small gift made a donation, and 21% of those receiving a large gift made a donation. (These percentages can be compared to 16% making donations among the expected counts.) So, it seems that the most effective strategy is to include a large gift, with the small gift making very little difference compared to no gift at all.
15.21: (a) Using the five-step process (HMC$^3$):

**Hypotheses (H):**

We want to determine if there is convincing evidence that field of study and smoking status are not independent.

The hypotheses of interest are:

\[ H_0: \text{field of study and smoking status are independent} \]

\[ H_a: \text{field of study and smoking status are not independent} \]

**Method (M):**

The purpose of this study is to compare two categorical variables (field of study and smoking status) from one population (college students at the University of Minnesota), so a chi-square test of independence will be considered.

The test statistic for this test is

\[ X^2 = \sum_{\text{all categories}} \frac{(\text{observed count} - \text{expected count})^2}{\text{expected count}} \]

When the null hypothesis is true, this statistic has approximately a chi-square distribution with (number of rows – 1)(number of columns – 1) degrees of freedom.

A significance level of \( \alpha = 0.01 \) will be used for this test.

**Check (C):**

We are told that a large random sample of students at the University of Minnesota was taken. The Minitab output in the textbook shows that all expected counts are at least 5. Because the sample was randomly selected and all expected counts are at least 5, the chi-square test of independence is appropriate.
Calculations (C):

Test statistic:

\[ X^2 = \sum_{\text{all categories}} \frac{(\text{observed count} - \text{expected count})^2}{\text{expected count}} \]

\[ = \frac{(176 - 189.23)^2}{189.23} + \frac{(489 - 475.77)^2}{475.77} + \cdots + \frac{(134 - 112.12)^2}{112.12} + \frac{(260 - 281.88)^2}{281.88} \]

\[ = 0.925 + 0.368 + \cdots + 4.272 + 1.699 \]

\[ = 90.853 \]

Degrees of freedom: \[ df = (\text{number of rows} - 1)(\text{number of columns} - 1) = (8)(1) = 8 \]

\( P \)-value: The \( P \)-value is the area under the chi-square curve with 8 degrees of freedom and to the right of 90.853. Therefore, the \( P \)-value is equal to \( P(X^2 \geq 90.853) \approx 0 \).

Communicate Results (C):

Because the \( P \)-value of approximately 0 is smaller than the specified significance level of \( \alpha = 0.01 \), we reject \( H_0 \). There is convincing evidence that the variables field of study and smoking status are not independent.

(b) The particularly high contributions to the chi-square statistic (in order of importance) come from the field of communication and languages, in which there was a disproportionately high number of smokers; from the field of mathematics, engineering, and sciences, in which there was a disproportionately low number of smokers; and from the field of social science and human services, in which there was a disproportionately high number of smokers.

15.22: Using the five-step process (HMC\(^3\)):

Hypotheses (H):

We want to determine if there is convincing evidence that locus of control and compulsive buyer behavior are associated (not independent).

The hypotheses of interest are:

\( H_0 \): locus of control and compulsive buyer behavior are independent

\( H_a \): locus of control and compulsive buyer behavior are not independent
Method (M):

The purpose of this study is to compare two categorical variables (locus of control and compulsive buyer behavior) from one population (college students at Midwestern public universities), so a chi-square test of association (independence) will be considered.

The test statistic for this test is

\[ X^2 = \sum_{\text{all categories}} \frac{(\text{observed count} - \text{expected count})^2}{\text{expected count}} \]

When the null hypothesis is true, this statistic has approximately a chi-square distribution with \((\text{number of rows} - 1)(\text{number of columns} - 1)\) degrees of freedom.

A significance level of \( \alpha = 0.01 \) will be used for this test.

Check (C):

We are told to assume that the sample was representative of college students at Midwestern public universities. The table below shows observed counts and, in parentheses, expected counts. Note that all expected counts are at least 5.

<table>
<thead>
<tr>
<th>Locus of Control</th>
<th>Compulsive Buyer?</th>
<th>Yes</th>
<th>3 (7.42)</th>
<th>14 (9.58)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Internal</td>
<td>No</td>
<td>52 (47.58)</td>
<td>57 (61.42)</td>
<td></td>
</tr>
<tr>
<td>External</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Because the sample was representative of the population and all expected counts are at least 5, the chi-square test of association (independence) is appropriate.

Calculations (C):

Test statistic:

\[ X^2 = \sum_{\text{all categories}} \frac{(\text{observed count} - \text{expected count})^2}{\text{expected count}} \]

\[ = \frac{(3 - 7.42)^2}{7.42} + \frac{(14 - 9.58)^2}{9.58} + \frac{(52 - 47.58)^2}{47.58} + \frac{(57 - 61.42)^2}{61.42} \]

\[ = 2.633 + 2.040 + 0.411 + 0.318 \]

\[ = 5.402 \]

Degrees of freedom: \( \text{df} = (\text{number of rows} - 1)(\text{number of columns} - 1) = (1)(1) = 1 \)
**P-value:** The *P*-value is the area under the chi-square curve with 1 degree of freedom and to the right of 5.402. Therefore, the *P*-value is equal to \( P(X^2 \geq 5.402) = 0.020 \).

Communicate Results (C):

Because the *P*-value of 0.020 is greater than the specified significance level of \( \alpha = 0.01 \), we fail to reject \( H_0 \). There is not convincing evidence that there is an association between locus of control and compulsive buyer behavior.

**Section 15.2 Exercise Set 2**

15.23: (a) Using the five-step process (HMC³):

**Hypotheses (H):**

We want to determine if there is convincing evidence that the proportions who would fall into each of the two response categories of usually eating 3 meals a day or rarely eating three meals a day are not the same for males and females.

The hypotheses of interest are:

\( H_0: \) proportions are the same for males and females

\( H_a: \) proportions are not the same for males and females

**Method (M):**

The purpose of this study is to compare two response categories (usually or rarely eat three meals a day) from two populations (male and female students), so a chi-square test of homogeneity will be considered.

The test statistic for this test is

\[
X^2 = \sum_{\text{all categories}} \frac{(\text{observed count} - \text{expected count})^2}{\text{expected count}}
\]

When the null hypothesis is true, this statistic has approximately a chi-square distribution with (number of rows – 1)(number of columns – 1) degrees of freedom.

A significance level of \( \alpha = 0.05 \) will be used for this test.
Check (C):

We are told that the students in each sample were randomly selected. The table below shows observed counts and, in parentheses, expected counts. Note that all expected counts are at least 5.

<table>
<thead>
<tr>
<th></th>
<th>Usually Eat 3 Meals a Day</th>
<th>Rarely Eat 3 Meals a Day</th>
</tr>
</thead>
<tbody>
<tr>
<td>Male</td>
<td>26 (21.76)</td>
<td>22 (26.24)</td>
</tr>
<tr>
<td>Female</td>
<td>37 (41.24)</td>
<td>54 (49.76)</td>
</tr>
</tbody>
</table>

Because the subjects were randomly selected and all expected counts are at least 5, the chi-square test of homogeneity is appropriate.

Calculations (C):

Test statistic:

\[
X^2 = \sum \frac{(\text{observed count} - \text{expected count})^2}{\text{expected count}}
\]

\[
= \frac{(26 - 21.76)^2}{21.76} + \frac{(22 - 26.24)^2}{26.24} + \frac{(37 - 41.24)^2}{41.24} + \frac{(54 - 49.76)^2}{49.76}
\]

\[
= 0.828 + 0.686 + 0.437 + 0.362 = 2.314
\]

Degrees of freedom: \(df = (\text{number of rows} - 1)(\text{number of columns} - 1) = (1)(1) = 1\)

\(P\)-value: The \(P\)-value is the area under the chi-square curve with 1 degree of freedom and to the right of 2.314. Therefore, the \(P\)-value is equal to \(P(X^2 \geq 2.314) = 0.128\).

Communicate Results (C):

Because the \(P\)-value of 0.128 is larger than the specified significance level of \(\alpha = 0.05\), we fail to reject \(H_0\). There is no convincing evidence that the proportions who would fall into each of the two response categories of usually eating 3 meals a day or rarely eating three meals a day are not the same for males and females.

(b) Yes, the calculations and conclusions from part (a) are consistent with the Mintab output. The expected counts, \(X^2\) statistic and \(P\)-value are the same as what is shown in part (a). Because the \(P\)-value is the same as in part (a), we would again fail to reject the null hypothesis.
(c) The two-sample $z$ test leads to the same conclusion. In this case, the $P$-value of 0.128 is greater than the significance level of $\alpha = 0.05$, so we fail to reject the null hypothesis. There is insufficient evidence to conclude that the proportion who usually eat 3 meals a day for males is different from the proportion who usually eat 3 meals a day for females.

(d) The $P$-values in parts (a) and (c) are exactly the same. This is not surprising because, since the analysis can be done using either the test of homogeneity or the two-sample $z$ test, the same data should yield the same conclusion.

15.24: (a) Using the five-step process (HMC$^3$):

Hypotheses (H):

We want to determine if there is convincing evidence that the proportions who would fall into each of the three hormone use categories are not the same for women who have been diagnosed with venous thrombosis and those who have not.

The hypotheses of interest are:

$H_0$: hormone use proportions are the same for women who have and have not been diagnosed with venous thrombosis

$H_a$: hormone use proportions are not the same for women who have and have not been diagnosed with venous thrombosis

Method (M):

The purpose of this study is to compare three response categories (the current hormone use) from two populations (women with and without a venous thrombosis diagnosis), so a chi-square test of homogeneity will be considered.

The test statistic for this test is

$$
\chi^2 = \sum_{\text{all categories}} \frac{(\text{observed count} - \text{expected count})^2}{\text{expected count}}
$$

When the null hypothesis is true, this statistic has approximately a chi-square distribution with $(\text{number of rows} - 1)(\text{number of columns} - 1)$ degrees of freedom.

A significance level of $\alpha = 0.05$ will be used for this test.

Check (C):

We are told that the women in each sample were randomly selected from the population of patients at a large HMO in Washington state. The table below shows...
observed counts and, in parentheses, expected counts. Note that all expected counts are at least 5.

<table>
<thead>
<tr>
<th>Current Hormone Use</th>
<th>None</th>
<th>Esterified Estrogen</th>
<th>Conjugated Equine Estrogen</th>
</tr>
</thead>
<tbody>
<tr>
<td>Venous Thrombosis</td>
<td>372 (371.57)</td>
<td>86 (123.31)</td>
<td>121 (84.12)</td>
</tr>
<tr>
<td>No Venous Thrombosis</td>
<td>1,439 (1,439.43)</td>
<td>515 (477.69)</td>
<td>289 (325.88)</td>
</tr>
</tbody>
</table>

Because the women were randomly selected and all expected counts are at least 5, the chi-square test of homogeneity is appropriate.

Calculations (C):

Test statistic:

\[
X^2 = \sum_{\text{all categories}} \frac{(\text{observed count} - \text{expected count})^2}{\text{expected count}}
\]

\[
= \frac{(372 - 371.57)^2}{371.57} + \frac{(86 - 123.31)^2}{123.31} + \cdots + \frac{(515 - 477.69)^2}{477.69} + \frac{(289 - 325.88)^2}{325.88}
\]

\[
= 0.0005 + 11.289 + \cdots + 2.914 + 4.173
\]

\[
= 34.544
\]

Degrees of freedom: \( df = (\text{number of rows} - 1)(\text{number of columns} - 1) = (1)(2) = 2 \)

\( P \)-value: The \( P \)-value is the area under the chi-square curve with 2 degrees of freedom and to the right of 34.544. Therefore, the \( P \)-value is equal to \( P(X^2 \geq 34.544) \approx 0 \).

Communicate Results (C):

Because the \( P \)-value of approximately 0 is smaller than the specified significance level of \( \alpha = 0.05 \), we reject \( H_0 \). There is convincing evidence that the proportions who would fall into each of the three hormone use categories are not the same for women who have been diagnosed with venous thrombosis and those who have not.

(b) It is reasonable to generalize the conclusion from the test in part (a) to the populations of women with and without a venous thrombosis diagnosis from the large HMO in the state of Washington. We can generalize conclusions to the populations from which the study participants were randomly selected.
15.25: Using the five-step process (HMC$^3$):

**Hypotheses (H):**

We want to determine if there is convincing evidence that there is an association between class standing and body art category.

The hypotheses of interest are:

- $H_0$: There is no association between class standing and body art category
- $H_a$: There is an association between class standing and body art category

**Method (M):**

The purpose of this study is to compare two categorical variables (class standing and body art category) from one population (students at this particular university), so a chi-square test of association will be considered.

The test statistic for this test is

$$X^2 = \sum_{\text{all categories}} \frac{(\text{observed count} - \text{expected count})^2}{\text{expected count}}$$

When the null hypothesis is true, this statistic has approximately a chi-square distribution with (number of rows – 1)(number of columns – 1) degrees of freedom.

A significance level of $\alpha = 0.01$ will be used for this test.

**Check (C):**

We are told that the sample of students is representative of all students at this particular university. The table below shows observed counts and, in parentheses, expected counts. Note that all expected counts are at least 5.

<table>
<thead>
<tr>
<th>Class</th>
<th>Body Piercings Only</th>
<th>Tattoos Only</th>
<th>Both Body Piercings and Tattoos</th>
<th>No Body Art</th>
</tr>
</thead>
<tbody>
<tr>
<td>Freshman</td>
<td>61 (49.71)</td>
<td>7 (15.09)</td>
<td>14 (18.51)</td>
<td>86 (84.69)</td>
</tr>
<tr>
<td>Sophomore</td>
<td>43 (37.88)</td>
<td>11 (11.49)</td>
<td>10 (14.11)</td>
<td>64 (64.52)</td>
</tr>
<tr>
<td>Junior</td>
<td>20 (23.38)</td>
<td>9 (7.09)</td>
<td>7 (8.71)</td>
<td>43 (39.82)</td>
</tr>
<tr>
<td>Senior</td>
<td>21 (34.03)</td>
<td>17 (10.33)</td>
<td>23 (12.67)</td>
<td>54 (57.97)</td>
</tr>
</tbody>
</table>
Because the sample was randomly selected and all expected counts are at least 5, the chi-square test of association is appropriate.

Calculations (C):

Test statistic:

\[ X^2 = \sum_{\text{all categories}} \frac{(\text{observed count} - \text{expected count})^2}{\text{expected count}} \]

\[ = \frac{(61 - 49.71)^2}{49.71} + \frac{(7 - 15.09)^2}{15.09} + \cdots + \frac{(23 - 12.67)^2}{12.67} + \frac{(54 - 57.97)^2}{57.97} \]

\[ = 2.562 + 4.334 + \cdots + 8.414 + 0.272 \]

\[ = 29.507 \]

Degrees of freedom: \( df = (\text{number of rows} - 1)(\text{number of columns} - 1) = (3)(3) = 9 \)

P-value: The P-value is the area under the chi-square curve with 9 degrees of freedom and to the right of 29.507. Therefore, the P-value is equal to \( P(X^2 \geq 29.507) = 0.001 \).

Communicate Results (C):

Because the P-value of 0.001 is less than the specified significance level of \( \alpha = 0.01 \), we reject \( H_0 \). There is convincing evidence that there is an association between class standing and body art category.

15.26: Using the five-step process (HMC\(^3\)):

Hypotheses (H):

We want to determine if there is convincing evidence of an association between age group and candidate selected in the election.

The hypotheses of interest are:

\( H_0: \) There is no association between age group and candidate selected in the election

\( H_a: \) There is an association between age group and candidate selected in the election
Method (M):

The purpose of this study is to compare two categorical variables (age group and candidate selected in the election) from one population (voters in the 2012 U.S. presidential election), so a chi-square test of association will be considered.

The test statistic for this test is

\[ X^2 = \sum_{\text{all categories}} \frac{(\text{observed count} - \text{expected count})^2}{\text{expected count}} \]

When the null hypothesis is true, this statistic has approximately a chi-square distribution with \((\text{number of rows} - 1)(\text{number of columns} - 1)\) degrees of freedom.

A significance level of \( \alpha = 0.05 \) will be used for this test.

Check (C):

We are told that this sample is representative of voters in the 2012 U.S. presidential election. The table below shows observed counts and, in parentheses, expected counts. Note that all expected counts are at least 5.

<table>
<thead>
<tr>
<th>Age Group</th>
<th>Obama</th>
<th>Romney</th>
<th>Other</th>
</tr>
</thead>
<tbody>
<tr>
<td>18-29</td>
<td>600 (510.0)</td>
<td>360 (467.5)</td>
<td>40 (22.5)</td>
</tr>
<tr>
<td>30-44</td>
<td>520 (510.0)</td>
<td>450 (467.5)</td>
<td>30 (22.5)</td>
</tr>
<tr>
<td>45-64</td>
<td>480 (510.0)</td>
<td>510 (467.5)</td>
<td>10 (22.5)</td>
</tr>
<tr>
<td>65 and older</td>
<td>440 (510.0)</td>
<td>550 (467.5)</td>
<td>10 (22.5)</td>
</tr>
</tbody>
</table>

Because the sample was representative of the population and all expected counts are at least 5, the chi-square test of association is appropriate.

Calculations (C):

Test statistic:

\[ X^2 = \sum_{\text{all categories}} \frac{(\text{observed count} - \text{expected count})^2}{\text{expected count}} \]

\[ = \frac{(600 - 510.0)^2}{510.0} + \frac{(360 - 467.5)^2}{467.5} + \cdots + \frac{(550 - 467.5)^2}{467.5} + \frac{(10 - 22.5)^2}{22.5} \]

\[ = 15.882 + 24.719 + \cdots + 14.559 + 6.944 \]

\[ = 101.248 \]

Degrees of freedom: \( df = (\text{number of rows} - 1)(\text{number of columns} - 1) = (3)(2) = 6 \)
**P-value:** The P-value is the area under the chi-square curve with 6 degrees of freedom and to the right of 101.248. Therefore, the P-value is equal to \( P(X^2 \geq 101.248) \approx 0 \).

**Communicate Results (C):**

Because the P-value of approximately 0 is less than the specified significance level of \( \alpha = 0.05 \), we reject \( H_0 \). There is convincing evidence that there is an association between age group and candidate selected in the election.

### Section 15.2 Additional Exercises

**15.27:** Answers may vary; one example follows. A political science researcher might be interested in seeing if there is evidence that the distribution of self-described political philosophy (liberal, moderate, conservative) among registered voters has changed over the years 2008, 2009, 2010, and 2011. Therefore, the populations that would be sampled are registered voters in each of the years 2008, 2009, 2010, and 2011. The variable that would be recorded is political philosophy (liberal, moderate, or conservative).

**15.28:** Answers may vary; one example follows. A medical researcher wants to know if there is an association between education level and survival after a heart attack among residents of Chicago. The population that would be sampled from is all residents of Chicago. The two variables that would be recorded are education level (not completed high school, completed high school but no college, or college/professional school completed) and survival status (survived or died).

**15.29:** In a chi-square goodness-of-fit test, one population is compared to fixed category proportions. In a chi-square test of homogeneity, two or more populations are compared.

**15.30:** In a chi-square test of homogeneity, two or more populations or treatments are being compared on the basis of one categorical variable. In a chi-square test of independence, the association between two categorical variables in a single population is being investigated.

**15.31:** Using the five-step process (HMC³):

**Hypotheses (H):**

We want to determine if there is convincing evidence that the use of ART and whether or not a baby is premature are associated (not independent).
The hypotheses of interest are:

\( H_0: \) There is no association between use of ART and whether or not a baby is premature

\( H_\alpha: \) There is an association between use of ART and whether or not a baby is premature

Method (M):

The purpose of this study is to compare two categorical variables (use of ART and whether or not a baby is premature) from one population (births), so a chi-square test of association (independence) will be considered.

The test statistic for this test is

\[ X^2 = \sum_{\text{all categories}} \frac{(\text{observed count} - \text{expected count})^2}{\text{expected count}} \]

When the null hypothesis is true, this statistic has approximately a chi-square distribution with (number of rows – 1)(number of columns – 1) degrees of freedom.

A significance level of \( \alpha = 0.01 \) will be used for this test.

Check (C):

We are told that a large random sample of babies was taken. The table below shows observed counts and, in parentheses, expected counts. Note that all expected counts are at least 5.

<table>
<thead>
<tr>
<th></th>
<th>Conceived using ART</th>
<th>Conceived Naturally</th>
</tr>
</thead>
<tbody>
<tr>
<td>Premature</td>
<td>154 (43.33)</td>
<td>2,405 (2,515.67)</td>
</tr>
<tr>
<td>Not Premature</td>
<td>212 (322.67)</td>
<td>18.843 (18,732.33)</td>
</tr>
</tbody>
</table>

Because the sample of births was randomly selected and all expected counts are at least 5, the chi-square test of association (independence) is appropriate.
Calculations (C):

Test statistic:

\[ X^2 = \sum_{\text{all categories}} \frac{(\text{observed count} - \text{expected count})^2}{\text{expected count}} \]

\[ = \frac{(154 - 43.33)^2}{43.33} + \frac{(2,405 - 2,515.67)^2}{2,515.67} + \frac{(212 - 322.67)^2}{322.67} + \frac{(18,843 - 18,732.33)^2}{18,732.33} \]

\[ = 282.632 + 4.868 + 37.956 + 0.654 \]

\[ = 326.111 \]

Degrees of freedom: \( df = (\text{number of rows} - 1)(\text{number of columns} - 1) = (1)(1) = 1 \)

\( P \)-value: The \( P \)-value is the area under the chi-square curve with 1 degree of freedom and to the right of 326.111. Therefore, the \( P \)-value is equal to \( P(X^2 \geq 326.111) \approx 0 \).

Communicate Results (C):

Because the \( P \)-value of approximately 0 is less than the specified significance level of \( \alpha = 0.01 \), we reject \( H_0 \). There is convincing evidence that there is an association between use of ART and whether or not a baby is premature.

15.32: (a)

<table>
<thead>
<tr>
<th>Country</th>
<th>Male</th>
<th>Female</th>
</tr>
</thead>
<tbody>
<tr>
<td>United States</td>
<td>550</td>
<td>450</td>
</tr>
<tr>
<td>Spain</td>
<td>195</td>
<td>305</td>
</tr>
<tr>
<td>Italy</td>
<td>190</td>
<td>310</td>
</tr>
<tr>
<td>India</td>
<td>200</td>
<td>800</td>
</tr>
</tbody>
</table>

(b) Using the five-step process (HMC³):

Hypotheses (H):

We want to determine if there is convincing evidence that the proportions of the genders of smartphone users are not the same for all four countries.

The hypotheses of interest are:

\( H_0: \) gender proportions are the same for all four countries

\( H_a: \) gender proportions are not the same for all four countries
Method (M):

The purpose of this study is to compare two response categories (gender) from four populations (the countries), so a chi-square test of homogeneity will be considered.

The test statistic for this test is

\[ X^2 = \sum_{\text{all categories}} \frac{(\text{observed count} - \text{expected count})^2}{\text{expected count}} \]

When the null hypothesis is true, this statistic has approximately a chi-square distribution with (number of rows – 1)(number of columns – 1) degrees of freedom.

A significance level of \( \alpha = 0.05 \) will be used for this test.

Check (C):

We are told that the samples from each country are representative of young people ages 15 to 24. The table below shows observed counts and, in parentheses, expected counts. Note that all expected counts are at least 5.

<table>
<thead>
<tr>
<th>Country</th>
<th>Male</th>
<th>Female</th>
</tr>
</thead>
<tbody>
<tr>
<td>United States</td>
<td>550 (378.33)</td>
<td>450 (621.67)</td>
</tr>
<tr>
<td>Spain</td>
<td>195 (189.17)</td>
<td>305 (310.83)</td>
</tr>
<tr>
<td>Italy</td>
<td>190 (189.17)</td>
<td>310 (310.83)</td>
</tr>
<tr>
<td>India</td>
<td>200 (378.33)</td>
<td>800 (621.67)</td>
</tr>
</tbody>
</table>

Because the samples are representative and all expected counts are at least 5, the chi-square test of homogeneity is appropriate.

Calculations (C):

Test statistic:

\[ X^2 = \sum_{\text{all categories}} \frac{(\text{observed count} - \text{expected count})^2}{\text{expected count}} \]

\[ = \frac{(550 - 378.33)^2}{378.33} + \frac{(450 - 621.67)^2}{621.67} + \cdots + \frac{(200 - 378.33)^2}{378.33} + \frac{(800 - 621.67)^2}{621.67} \]

\[ = 77.893 + 47.404 + \cdots + 84.060 + 51.157 \]

\[ = 260.809 \]

Degrees of freedom: \( df = (\text{number of rows} - 1)(\text{number of columns} - 1) = 3(1) = 3 \)
P-value: The P-value is the area under the chi-square curve with 3 degrees of freedom and to the right of 260.809. Therefore, the P-value is equal to \( P(X^2 \geq 260.809) \approx 0 \).

Communicate Results (C):

Because the P-value of approximately 0 is smaller than the specified significance level of \( \alpha = 0.05 \), we reject \( H_0 \). There is convincing evidence that the proportions of the genders of smart phone users are not the same for all four countries.

15.33: Using the five-step process (HMC³):

Hypotheses (H):

We want to determine if there is convincing evidence that the proportions in the three candidate categories are not the same for all three of the states.

The hypotheses of interest are:

\begin{align*}
H_0: & \quad \text{The proportions in the three candidate categories are the same for all three of the states} \\
H_a: & \quad \text{The proportions in the three candidate categories are not the same for all three of the states}
\end{align*}

Method (M):

The purpose of this study is to compare one categorical variable (candidate) from three populations (states), so a chi-square test of homogeneity will be considered.

The test statistic for this test is

\[
X^2 = \sum_{\text{all categories}} \frac{(\text{observed count} - \text{expected count})^2}{\text{expected count}}
\]

When the null hypothesis is true, this statistic has approximately a chi-square distribution with \((\text{number of rows} - 1)(\text{number of columns} - 1)\) degrees of freedom.

A significance level of \( \alpha = 0.01 \) will be used for this test.
Check (C):

We are told that the samples are representative of voters from the three states in the 2012 U.S. presidential election. The table below shows observed counts and, in parentheses, expected counts. Note that all expected counts are at least 5.

<table>
<thead>
<tr>
<th>State</th>
<th>Candidate</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Obama</td>
<td>Romney</td>
<td>Other</td>
</tr>
<tr>
<td>Ohio</td>
<td>500 (503.33)</td>
<td>490 (483.33)</td>
<td>10 (13.33)</td>
</tr>
<tr>
<td>Texas</td>
<td>430 (503.33)</td>
<td>550 (483.33)</td>
<td>20 (13.33)</td>
</tr>
<tr>
<td>New York</td>
<td>580 (503.33)</td>
<td>410 (483.33)</td>
<td>10 (13.33)</td>
</tr>
</tbody>
</table>

Because the samples are representative of the populations of voters from the three states and all expected counts are at least 5, the chi-square test of homogeneity is appropriate.

Calculations (C):

Test statistic:

\[ X^2 = \sum \frac{(\text{observed count} - \text{expected count})^2}{\text{expected count}} \]

\[ = \frac{(500 - 503.33)^2}{503.33} + \frac{(490 - 483.33)^2}{483.33} + \cdots + \frac{(10 - 13.33)^2}{13.33} \]

\[ = 0.022 + 0.092 + \cdots + 11.126 + 0.833 \]

\[ = 47.798 \]

Degrees of freedom: \( df = (\text{number of rows} - 1)(\text{number of columns} - 1) = (2)(2) = 4 \)

\( P \)-value: The \( P \)-value is the area under the chi-square curve with 4 degrees of freedom and to the right of 47.798. Therefore, the \( P \)-value is equal to \( P(X^2 \geq 47.798) \approx 0 \).

Communicate Results (C):

Because the \( P \)-value of approximately 0 is less than the specified significance level of \( \alpha = 0.01 \), we reject \( H_0 \). There is convincing evidence that the proportions in the three candidate categories are not the same for all three of the states.
15.34: Using the five-step process (HMC³):

Hypotheses (H):

We want to determine if there is convincing evidence that the proportion of correct sex identifications differs for the three different nose views.

The hypotheses of interest are:

\( H_0: \) The proportion of correct sex identifications does not differ (are the same) for the three different nose views

\( H_a: \) The proportion of correct sex identifications differs (are not the same) for the three different nose views

Method (M):

The purpose of this study is to compare one categorical variable (sex identification) from three populations (nose view – front, profile, or three-quarter), so a chi-square test of homogeneity will be considered.

The test statistic for this test is

\[
X^2 = \sum_{\text{all categories}} \frac{(\text{observed count} - \text{expected count})^2}{\text{expected count}}
\]

When the null hypothesis is true, this statistic has approximately a chi-square distribution with \((\text{number of rows} - 1)(\text{number of columns} - 1)\) degrees of freedom.

A significance level of \( \alpha = 0.05 \) will be used for this test.

Check (C):

We are not told that students were randomly assigned to one of the treatments (different nose view), but we will assume that is the case. The table below shows observed counts and, in parentheses, expected counts. Note that all expected counts are at least 5.

<table>
<thead>
<tr>
<th>Sex ID</th>
<th>Nose View</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Front</td>
<td>Profile</td>
<td>Three-Quarter</td>
<td></td>
</tr>
<tr>
<td>Correct</td>
<td>23 (26)</td>
<td>26 (26)</td>
<td>29 (26)</td>
<td></td>
</tr>
<tr>
<td>Incorrect</td>
<td>17 (14)</td>
<td>14 (14)</td>
<td>11 (14)</td>
<td></td>
</tr>
</tbody>
</table>
Because we are assuming the students were randomly assigned to the different nose views and all expected counts are at least 5, the chi-square test of homogeneity is appropriate.

Calculations (C):

Test statistic:

\[ X^2 = \sum_{\text{all categories}} \frac{(\text{observed count} - \text{expected count})^2}{\text{expected count}} \]

\[ = \frac{(23 - 26)^2}{26} + \frac{(26 - 26)^2}{26} + \frac{(29 - 26)^2}{26} \]
\[ + \frac{(17 - 14)^2}{14} + \frac{(14 - 14)^2}{14} + \frac{(11 - 14)^2}{14} \]
\[ = 0.346 + 0 + 0.346 + 0.643 + 0 + 0.643 \]
\[ = 1.978 \]

Degrees of freedom: \( df = (\text{number of rows} - 1)(\text{number of columns} - 1) = (1)(2) = 2 \)

\( P \)-value: The \( P \)-value is the area under the chi-square curve with 2 degrees of freedom and to the right of 1.978. Therefore, the \( P \)-value is equal to \( P(X^2 \geq 1.978) = 0.372 \).

Communicate Results (C):

Because the \( P \)-value of 0.372 is greater than the specified significance level of \( \alpha = 0.05 \), we fail to reject \( H_0 \). There is not convincing evidence that the proportion of correct sex identifications differs for the three different nose views.

15.35: Using the five-step process (HMC^3):

Hypotheses (H):

We want to determine if there is convincing evidence that city of residence and vehicle type are associated.

The hypotheses of interest are:

\( H_0: \) There is no association between city of residence and vehicle type

\( H_a: \) There is an association between city of residence and vehicle type
Method (M):

The purpose of this study is to compare two categorical variables (city of residence and vehicle type) from one population (individuals in the San Francisco bay area), so a chi-square test of association will be considered.

The test statistic for this test is

\[ X^2 = \sum_{\text{all categories}} \frac{(\text{observed count} - \text{expected count})^2}{\text{expected count}} \]

When the null hypothesis is true, this statistic has approximately a chi-square distribution with (number of rows – 1)(number of columns – 1) degrees of freedom.

A significance level of \( \alpha = 0.05 \) will be used for this test.

Check (C):

We are told to assume that the sample is a random sample of San Francisco Bay area residents. The table below shows observed counts and, in parentheses, expected counts. Note that all expected counts are at least 5.

<table>
<thead>
<tr>
<th>Vehicle Type</th>
<th>Concord</th>
<th>Pleasant Hills</th>
<th>North San Francisco</th>
</tr>
</thead>
<tbody>
<tr>
<td>Small</td>
<td>68 (89.06)</td>
<td>83 (107.02)</td>
<td>221 (175.92)</td>
</tr>
<tr>
<td>Compact</td>
<td>63 (56.74)</td>
<td>68 (68.18)</td>
<td>106 (112.08)</td>
</tr>
<tr>
<td>Midsize</td>
<td>88 (84.51)</td>
<td>123 (101.55)</td>
<td>142 (166.94)</td>
</tr>
<tr>
<td>Large</td>
<td>24 (12.69)</td>
<td>18 (15.25)</td>
<td>11 (25.06)</td>
</tr>
</tbody>
</table>

Because we were told to assume the sample was randomly selected and all expected counts are at least 5, the chi-square test of association is appropriate.
Calculations (C):

Test statistic:

\[ X^2 = \sum \frac{(\text{observed count} - \text{expected count})^2}{\text{expected count}} \]

\[ = \frac{(68 - 89.06)^2}{89.06} + \frac{(83 - 107.02)^2}{107.02} + \cdots + \frac{(18 - 15.25)^2}{15.25} + \frac{(11 - 25.06)^2}{25.06} \]

\[ = 4.980 + 5.391 + \cdots + 0.497 + 7.892 \]

\[ = 49.813 \]

Degrees of freedom: \( df = (\text{number of rows} - 1)(\text{number of columns} - 1) = (3)(2) = 6 \)

\( P \)-value: The \( P \)-value is the area under the chi-square curve with 6 degrees of freedom and to the right of 49.813. Therefore, the \( P \)-value is equal to \( P(X^2 \geq 49.813) \approx 0 \).

Communicate Results (C):

Because the \( P \)-value of approximately 0 is less than the specified significance level of \( \alpha = 0.05 \), we reject \( H_0 \). There is convincing evidence that there is an association between city of residence and vehicle type.

15.36: Using the five-step process (HMC\(^3\)):

Hypotheses (H):

We want to determine if there is convincing evidence that there is an association between position and role.

The hypotheses of interest are:

\( H_0 \): There is no association between position and role

\( H_a \): There is an association between position and role

Method (M):

The purpose of this study is to compare two categorical variables (position and role) from one population (lionesses), so a chi-square test of association will be considered.
The test statistic for this test is

\[ X^2 = \sum_{\text{all categories}} \frac{(\text{observed count} - \text{expected count})^2}{\text{expected count}} \]

When the null hypothesis is true, this statistic has approximately a chi-square distribution with \((\text{number of rows} - 1)(\text{number of columns} - 1)\) degrees of freedom.

A significance level of \( \alpha = 0.01 \) will be used for this test.

Check (C):

We are not told that the sample of lionesses was randomly selected, so we must assume this is the case. The table below shows observed counts and, in parentheses, expected counts. Note that all expected counts are at least 5.

<table>
<thead>
<tr>
<th>Role</th>
<th>Position</th>
<th>Initiate Chase</th>
<th>Participate in Chase</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Center</td>
<td>28 (39.04)</td>
<td>48 (36.96)</td>
</tr>
<tr>
<td></td>
<td>Wing</td>
<td>66 (54.96)</td>
<td>41 (52.04)</td>
</tr>
</tbody>
</table>

Because we are assuming that the sample was randomly selected and all expected counts are at least 5, the chi-square test of association is appropriate.

Calculations (C):

Test statistic:

\[ X^2 = \sum_{\text{all categories}} \frac{(\text{observed count} - \text{expected count})^2}{\text{expected count}} \]

\[ = \frac{(28 - 39.04)^2}{39.04} + \frac{(48 - 36.96)^2}{36.96} + \frac{(66 - 54.96)^2}{54.96} + \frac{(41 - 52.04)^2}{52.04} \]

\[ = 3.121 + 3.296 + 2.217 + 2.341 \]

\[ = 10.976 \]

Degrees of freedom: \( \text{df} = (\text{number of rows} - 1)(\text{number of columns} - 1) = (1)(1) = 1 \)

\( P \)-value: The \( P \)-value is the area under the chi-square curve with 1 degree of freedom and to the right of 10.976. Therefore, the \( P \)-value is equal to \( P(X^2 \geq 10.976) = 0.001 \).
Communicate Results (C):

Because the $P$-value of 0.001 is less than the specified significance level of $\alpha = 0.01$, we reject $H_0$. There is convincing evidence that there is an association between position and role.

The assumption about the sample that must be true for the chi-square test to be an appropriate way to analyze these data is that the sample must have been randomly selected or be representative of the population of lionesses.

15.37: Using the five-step process (HMC$^3$):

Hypotheses (H):

We want to determine if there is convincing evidence that age of children and parental response are associated.

The hypotheses of interest are:

$H_0$: There is no association between age of children and parental response

$H_a$: There is an association between age of children and parental response

Method (M):

The purpose of this study is to compare two categorical variables (age of children and parental response) from one population (parents of preteen and teenage children), so a chi-square test of association will be considered.

The test statistic for this test is

$$\chi^2 = \sum_{\text{all categories}} \frac{(\text{observed count} - \text{expected count})^2}{\text{expected count}}$$

When the null hypothesis is true, this statistic has approximately a chi-square distribution with (number of rows – 1)(number of columns – 1) degrees of freedom.

A significance level of $\alpha = 0.05$ will be used for this test.

Check (C):

We are only told that a telephone survey of parents with preteen and teenage children was taken, and not how the respondents were selected. We will assume the sample was selected randomly. The table below shows observed counts and, in parentheses, expected counts. Note that all expected counts are at least 5.
Because we are assuming the sample was randomly selected and all expected counts are at least 5, the chi-square test of association is appropriate.

Calculations (C):

Test statistic:

\[
X^2 = \sum \frac{(\text{observed count} - \text{expected count})^2}{\text{expected count}}
\]

\[
= \frac{(126 - 139.07)^2}{139.07} + \frac{(149 - 135.93)^2}{135.93} + \cdots + \frac{(51 - 38.94)^2}{38.94} + \frac{(26 - 38.06)^2}{38.06}
\]

\[
= 1.229 + 1.257 + \cdots + 3.735 + 3.821
\]

\[
= 10.091
\]

Degrees of freedom: \( df = (\text{number of rows} - 1) \times (\text{number of columns} - 1) = (2)(1) = 2 \)

\( P \)-value: The \( P \)-value is the area under the chi-square curve with 2 degrees of freedom and to the right of 10.091. Therefore, the \( P \)-value is equal to \( P(X^2 \geq 10.091) = 0.006 \).

Communicate Results (C):

Because the \( P \)-value of 0.006 is less than the specified significance level of \( \alpha = 0.05 \), we reject \( H_0 \). There is convincing evidence that there is an association between age of children and parental response.

15.38: Using the five-step process (HMC\(^3\)):

Hypotheses (H):

We want to determine if there is convincing evidence that there is an association between region of residence and response to the question “Sometimes it is necessary to discipline a child with a good, hard spanking.”

The hypotheses of interest are:

\( H_0 \): There is no association between region and response
\( H_0: \) There is an association between region and response

Method (M):

The purpose of this study is to compare two categorical variables (region and response) from one population (adults), so a chi-square test of association will be considered.

The test statistic for this test is

\[
\chi^2 = \sum_{\text{all categories}} \frac{(\text{observed count} - \text{expected count})^2}{\text{expected count}}
\]

When the null hypothesis is true, this statistic has approximately a chi-square distribution with \((\text{number of rows} - 1)(\text{number of columns} - 1)\) degrees of freedom.

A significance level of \( \alpha = 0.01 \) will be used for this test.

Check (C):

We are told that the sample of adults was randomly selected. The table below shows observed counts and, in parentheses, expected counts. Note that all expected counts are at least 5.

<table>
<thead>
<tr>
<th>Region</th>
<th>Agree</th>
<th>Disagree</th>
</tr>
</thead>
<tbody>
<tr>
<td>Northeast</td>
<td>130 (150.35)</td>
<td>59 (38.65)</td>
</tr>
<tr>
<td>West</td>
<td>146 (149.55)</td>
<td>42 (38.45)</td>
</tr>
<tr>
<td>Midwest</td>
<td>211 (209.22)</td>
<td>52 (53.78)</td>
</tr>
<tr>
<td>South</td>
<td>291 (268.88)</td>
<td>47 (69.12)</td>
</tr>
</tbody>
</table>

Because the sample was randomly selected and all expected counts are at least 5, the chi-square test of association is appropriate.
Calculations (C):

Test statistic:

\[ X^2 = \sum \frac{(\text{observed count} - \text{expected count})^2}{\text{expected count}} \]

\[ = \frac{(130 - 150.35)^2}{150.35} + \frac{(59 - 38.65)^2}{38.65} + \cdots + \frac{(291 - 268.88)^2}{268.88} + \frac{(47 - 69.12)^2}{69.12} \]

\[ = 2.754 + 10.714 + \cdots + 1.820 + 7.079 \]

\[ = 22.855 \]

Degrees of freedom: \( df = (\text{number of rows} - 1)(\text{number of columns} - 1) = (3)(1) = 3 \)

\( P \)-value: The \( P \)-value is the area under the chi-square curve with 3 degrees of freedom and to the right of 22.855. Therefore, the \( P \)-value is equal to \( P(X^2 \geq 22.855) \approx 0 \).

Communicate Results (C):

Because the \( P \)-value of approximately 0 is less than the specified significance level of \( \alpha = 0.01 \), we reject \( H_0 \). There is convincing evidence that there is an association between region of residence and response.

15.39: (a) Since the study was conducted using separate random samples of male and female inmates, this is a test for homogeneity.

(b) Using the five-step process (HMC³):

Hypotheses (H):

We want to determine if there is convincing evidence that the proportions of types of crime are not all the same for male and female inmates.

The hypotheses of interest are:

\( H_0 \): the proportions falling into the crime categories are the same for male and female inmates

\( H_a \): The proportions falling into the crime categories are not the same for male and female inmates

Method (M):

The purpose of this study is to compare genders on the basis of one categorical variable (type of crime), so a chi-square test of homogeneity will be considered.
The test statistic for this test is

\[ X^2 = \sum_{\text{all categories}} \frac{(\text{observed count} - \text{expected count})^2}{\text{expected count}} \]

When the null hypothesis is true, this statistic has approximately a chi-square distribution with \((\text{number of rows} - 1) \times (\text{number of columns} - 1)\) degrees of freedom.

A significance level of \(\alpha = 0.05\) will be used for this test.

Check (C):

We are told that two random samples (500 male inmates and 500 female inmates) were taken. The table below shows observed counts and, in parentheses, expected counts. Note that all expected counts are at least 5.

<table>
<thead>
<tr>
<th>Type of Crime</th>
<th>Gender</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Male</td>
<td>Female</td>
<td></td>
</tr>
<tr>
<td>Violent</td>
<td>117 (91.5)</td>
<td>66 (91.5)</td>
<td></td>
</tr>
<tr>
<td>Property</td>
<td>150 (155)</td>
<td>160 (155)</td>
<td></td>
</tr>
<tr>
<td>Drug</td>
<td>109 (138.5)</td>
<td>168 (138.5)</td>
<td></td>
</tr>
<tr>
<td>Public-order</td>
<td>124 (115)</td>
<td>106 (115)</td>
<td></td>
</tr>
</tbody>
</table>

Because the inmates were randomly selected and all expected counts are at least 5, the chi-square test of homogeneity is appropriate.

Calculations (C):

Test statistic:

\[ X^2 = \sum_{\text{all categories}} \frac{(\text{observed count} - \text{expected count})^2}{\text{expected count}} \]

\[ = \frac{(117 - 91.5)^2}{91.5} + \frac{(66 - 91.5)^2}{91.5} + \cdots + \frac{(124 - 115)^2}{115} + \frac{(106 - 115)^2}{115} \]

\[ = 7.107 + 7.107 + \cdots + 0.704 + 0.704 \]

\[ = 28.511 \]

Degrees of freedom: \(df = (\text{number of rows} - 1) \times (\text{number of columns} - 1) = (3)(1) = 3\)

\(P\)-value: The \(P\)-value is the area under the chi-square curve with 3 degrees of freedom and to the right of 28.511. Therefore, the \(P\)-value is equal to \(P(X^2 \geq 28.511) \approx 0\).
Communicate Results (C):

Because the $P$-value of approximately 0 is smaller than the specified significance level of $\alpha = 0.05$, we reject $H_0$. There is convincing evidence that the proportions falling into the crime categories are not all the same for male and female inmates.

Are You Ready To Move On? Chapter 15 Review Exercises

15.40: Using the five-step process (HMC$^3$):

Hypotheses (H):

We want to determine if there is convincing evidence that the proportions of characters of different ethnicities appearing in commercials are not the same as the census proportions.

Define the following population characteristics of interest:

\[ p_1 = \text{proportion of African American characters appearing in commercials} \]
\[ p_2 = \text{proportion of Asian characters appearing in commercials} \]
\[ p_3 = \text{proportion of Caucasian characters appearing in commercials} \]
\[ p_4 = \text{proportion of Hispanic characters appearing in commercials} \]

State the appropriate hypotheses:

\[ H_0 : p_1 = 0.177, \ p_2 = 0.032, \ p_3 = 0.734, \ p_4 = 0.057 \]
\[ H_a : \text{At least one of the population proportions is not as stated in } H_0 \]

Method (M):

Because the answers to the four key questions are (Q) hypothesis testing, (S) sample data, (T) one categorical variable with more than two categories, and (N) one sample, a chi-square goodness-of-fit test will be considered.

The test statistic for this test is

\[ X^2 = \sum_{\text{all categories}} \frac{(\text{observed count} - \text{expected count})^2}{\text{expected count}} \]

When the null hypothesis is true, this statistic has approximately a chi-square distribution with $k - 1$ degrees of freedom.
A significance level of $\alpha = 0.01$ will be used for this test.

Check (C):

We are not told if the data were randomly selected or representative of the population of commercials, so we must assume so to carry out this statistical test. The expected counts in each category are $404(0.177) = 71.508$ for the African American ethnicity group, $404(0.032) = 12.928$ for the Asian ethnicity group, $404(0.734) = 296.536$ for the Caucasian ethnicity group, and $404(0.057) = 23.028$ for the Hispanic ethnicity group, which are all greater than 5, so the sample size is large enough. Assuming that the data were collected properly, the two conditions necessary to use the chi-square goodness-of-fit test have been satisfied.

Calculations (C):

Test statistic:

$$X^2 = \sum_{\text{all categories}} \frac{(\text{observed count} - \text{expected count})^2}{\text{expected count}}$$

$$= \frac{(57 - 71.508)^2}{71.508} + \frac{(11 - 12.928)^2}{12.928} + \frac{(330 - 296.536)^2}{296.536} + \frac{(6 - 23.028)^2}{23.028}$$

$$= 2.9435 + 0.2875 + 3.7764 + 12.5913$$

$$= 19.5987$$

Degrees of freedom: $df = k - 1 = 4 - 1 = 3$

$P$-value: The $P$-value is the area under the chi-square curve with 3 degrees of freedom and to the right of 19.5987. Therefore, the $P$-value is equal to $P(X^2 \geq 19.5987) \approx 0$.

Communicate Results (C):

Because the $P$-value of approximately 0 is less than the chosen significance level of $\alpha = 0.01$, we reject $H_0$. There is convincing evidence that the proportions of characters of different ethnicities appearing in commercials are not the same as the census proportions.

15.41: Using the five-step process (HMC$^3$):

Hypotheses (H):

We want to determine if there is convincing evidence that fatal bicycle accidents are not equally likely to occur in each of the 3-hour time periods.
Define the following population characteristics of interest:

\[ p_1 = \text{proportion of fatal bicycle accidents from Midnight to 3 A.M.} \]
\[ p_2 = \text{proportion of fatal bicycle accidents from 3 A.M. to 6 A.M.} \]
\[ p_3 = \text{proportion of fatal bicycle accidents from 6 A.M. to 9 A.M.} \]
\[ p_4 = \text{proportion of fatal bicycle accidents from 9 A.M. to Noon} \]
\[ p_5 = \text{proportion of fatal bicycle accidents from Noon to 3 P.M.} \]
\[ p_6 = \text{proportion of fatal bicycle accidents from 3 P.M. to 6 P.M.} \]
\[ p_7 = \text{proportion of fatal bicycle accidents from 6 P.M. to 9 P.M.} \]
\[ p_8 = \text{proportion of fatal bicycle accidents from 9 P.M. to Midnight} \]

State the appropriate hypotheses:

\[ H_0 : p_1 = p_2 = p_3 = p_4 = p_5 = p_6 = p_7 = p_8 = \frac{1}{8} \]

\[ H_a : \text{At least one of the population proportions is not } \frac{1}{8} \]

Method (M):

Because the answers to the four key questions are (Q) hypothesis testing, (S) sample data, (T) one categorical variable with more than two categories, and (N) one sample, a chi-square goodness-of-fit test will be considered.

The test statistic for this test is

\[ X^2 = \sum_{\text{all categories}} \frac{(\text{observed count} - \text{expected count})^2}{\text{expected count}} \]

When the null hypothesis is true, this statistic has approximately a chi-square distribution with \( k - 1 \) degrees of freedom.

A significance level of \( \alpha = 0.05 \) will be used for this test.

Check (C):

We are told to assume these 715 bicycle accidents are a random sample of fatal bicycle accidents. In addition, all expected counts in each category are all equal to
\[
715 \left( \frac{1}{8} \right) = 89.375, \text{ which is greater than } 5, \text{ so the sample size is large enough. The two conditions necessary to use the chi-square goodness-of-fit test have been satisfied.}
\]

Calculations (C):

Test statistic:

\[
X^2 = \sum \frac{(\text{observed count} - \text{expected count})^2}{\text{expected count}}
\]

\[
= \frac{(38 - 89.375)^2}{89.375} + \frac{(29 - 89.375)^2}{89.375} + \cdots + \frac{(166 - 89.375)^2}{89.375} + \frac{(113 - 89.375)^2}{89.375}
\]

\[
= 29.5316 + 40.7848 + \cdots + 65.6939 + 6.2449
\]

\[
= 166.958
\]

Degrees of freedom: \( df = k - 1 = 8 - 1 = 7 \)

\( P \)-value: The \( P \)-value is the area under the chi-square curve with 7 degrees of freedom and to the right of 166.958. Therefore, the \( P \)-value is equal to \( P(X^2 \geq 166.958) \approx 0. \)

Communicate Results (C):

Because the \( P \)-value of approximately 0 is less than the chosen significance level of \( \alpha = 0.05 \), we reject \( H_0 \). There is convincing evidence that fatal accidents are not equally likely to occur in each of the eight time periods.

15.42: Using the five-step process (HMC):

Hypotheses (H):

We want to determine if there is convincing evidence that bicycle fatalities are not twice as likely to occur between noon and midnight as during midnight to noon.

Define the following population characteristics of interest:

\( p_1 = \) proportion of accidents occurring between midnight and noon

\( p_2 = \) proportion of accidents occurring between noon and midnight

State the appropriate hypotheses:

\( H_0 : p_1 = \frac{1}{3} \text{ and } p_2 = \frac{2}{3} \)


\( H_a \): The population proportions stated in \( H_0 \) are not correct

Method (M):

Because the answers to the four key questions are (Q) hypothesis testing, (S) sample data, (T) one categorical variable with two categories, and (N) one sample, a chi-square goodness-of-fit test will be considered.

The test statistic for this test is

\[
X^2 = \sum_{\text{all categories}} \frac{(\text{observed count} - \text{expected count})^2}{\text{expected count}}
\]

When the null hypothesis is true, this statistic has approximately a chi-square distribution with \( k - 1 \) degrees of freedom.

A significance level of \( \alpha = 0.05 \) will be used for this test.

Check (C):

We are told to assume these 715 bicycle accidents are a random sample of fatal bicycle accidents. In addition, the expected counts in the midnight to noon category is

\[
715 \left( \frac{1}{3} \right) = 238.333
\]

and the expected counts in the noon to midnight category is

\[
715 \left( \frac{2}{3} \right) = 476.667
\]

which are both greater than 5, so the sample size is large enough.

The two conditions necessary to use the chi-square goodness-of-fit test have been satisfied.

Calculations (C):

Test statistic:

\[
X^2 = \sum_{\text{all categories}} \frac{(\text{observed count} - \text{expected count})^2}{\text{expected count}}
\]

\[
= \frac{(210 - 238.333)^2}{238.333} + \frac{(505 - 476.667)^2}{476.667}
\]

\[
= 3.36829 + 1.68415
\]

\[
= 5.05244
\]

Degrees of freedom: \( df = k - 1 = 2 - 1 = 1 \)
P-value: The P-value is the area under the chi-square curve with 1 degree of freedom and to the right of 5.05244. Therefore, the P-value is equal to 
\[ P(X^2 \geq 5.05244) = 0.025. \]

Communicate Results (C):

Because the P-value of 0.025 is less than the chosen significance level of \( \alpha = 0.05 \), we reject \( H_0 \). There is convincing evidence that bicycle fatalities are not twice as likely to occur between noon and midnight as during midnight to noon, so the data provide evidence against the hypothesis.

15.43: (a) Using the five-step process (HMC3):

Hypotheses (H):

We want to determine if there is convincing evidence that fatal bicycle accidents are not equally likely to occur in each of the 12 months.

Define the following population characteristics of interest:

\[ p_1 = \text{proportion of fatal bicycle accidents in January} \]
\[ p_2 = \text{proportion of fatal bicycle accidents in February} \]
\[ p_3 = \text{proportion of fatal bicycle accidents in March} \]
\[ p_4 = \text{proportion of fatal bicycle accidents in April} \]
\[ p_5 = \text{proportion of fatal bicycle accidents in May} \]
\[ p_6 = \text{proportion of fatal bicycle accidents in June} \]
\[ p_7 = \text{proportion of fatal bicycle accidents in July} \]
\[ p_8 = \text{proportion of fatal bicycle accidents in August} \]
\[ p_9 = \text{proportion of fatal bicycle accidents in September} \]
\[ p_{10} = \text{proportion of fatal bicycle accidents in October} \]
\[ p_{11} = \text{proportion of fatal bicycle accidents in November} \]
\[ p_{12} = \text{proportion of fatal bicycle accidents in December} \]

State the appropriate hypotheses:
Method (M):

Because the answers to the four key questions are (Q) hypothesis testing, (S) sample data, (T) one categorical variable with more than two categories, and (N) one sample, a chi-square goodness-of-fit test will be considered.

The test statistic for this test is

\[ X^2 = \sum_{\text{all categories}} \frac{(\text{observed count} - \text{expected count})^2}{\text{expected count}} \]

When the null hypothesis is true, this statistic has approximately a chi-square distribution with \( k - 1 \) degrees of freedom.

A significance level of \( \alpha = 0.01 \) will be used for this test.

Check (C):

We are told (in exercise 15.41) to assume these bicycle accidents are a random sample of fatal bicycle accidents. In addition, all expected counts in each category are all equal to \( \frac{719}{12} = 59.9167 \), which is greater than 5, so the sample size is large enough. The two conditions necessary to use the chi-square goodness-of-fit test have been satisfied.

Calculations (C):

Test statistic:

\[
X^2 = \sum_{\text{all categories}} \frac{(\text{observed count} - \text{expected count})^2}{\text{expected count}}
\]

\[
= \frac{(38 - 59.9167)^2}{59.9167} + \frac{(32 - 59.9167)^2}{59.9167} + \cdots + \frac{(42 - 59.9167)^2}{59.9167} + \frac{(40 - 59.9167)^2}{59.9167}
\]

\[
= 8.0168 + 13.0071 + \cdots + 5.3576 + 6.6204
\]

\[
= 82.1627
\]

Degrees of freedom: \( df = k - 1 = 12 - 1 = 11 \)
\textit{P}-value: The $P$-value is the area under the chi-square curve with 11 degrees of freedom and to the right of 82.1627. Therefore, the $P$-value is equal to $P(X^2 \geq 82.1627) \approx 0$.
Communicate Results (C):

Because the $P$-value of approximately 0 is less than the chosen significance level of $\alpha = 0.01$, we reject $H_0$. There is convincing evidence that fatal accidents are not equally likely to occur in each of the months.

(b)

$$H_0 : p_1 = \frac{31}{366}, p_2 = \frac{29}{366}, p_3 = \frac{31}{366}, p_4 = \frac{30}{366}, p_5 = \frac{31}{366}, p_6 = \frac{30}{366},$$

$$p_7 = \frac{31}{366}, p_8 = \frac{31}{366}, p_9 = \frac{30}{366}, p_{10} = \frac{31}{366}, p_{11} = \frac{30}{366}, p_{12} = \frac{31}{366}$$

$H_a :$ At least one of the population proportions is not correct as written

(c) Using the five-step process (HMC$^3$):

Hypotheses (H):

We want to determine if there is convincing evidence that fatal bicycle accidents are not equally likely to occur in each of the 12 months in proportion to the lengths of the months.

Define the following population characteristics of interest:

$p_1 =$ proportion of fatal bicycle accidents in January

$p_2 =$ proportion of fatal bicycle accidents in February

$p_3 =$ proportion of fatal bicycle accidents in March

$p_4 =$ proportion of fatal bicycle accidents in April

$p_5 =$ proportion of fatal bicycle accidents in May

$p_6 =$ proportion of fatal bicycle accidents in June

$p_7 =$ proportion of fatal bicycle accidents in July

$p_8 =$ proportion of fatal bicycle accidents in August

$p_9 =$ proportion of fatal bicycle accidents in September

$p_{10} =$ proportion of fatal bicycle accidents in October

$p_{11} =$ proportion of fatal bicycle accidents in November
\( p_{12} = \text{proportion of fatal bicycle accidents in December} \)

State the appropriate hypotheses:

\[
H_0: p_1 = \frac{31}{366}, p_2 = \frac{29}{366}, p_3 = \frac{31}{366}, p_4 = \frac{30}{366}, p_5 = \frac{31}{366}, p_6 = \frac{30}{366}, \\
p_7 = \frac{31}{366}, p_8 = \frac{31}{366}, p_9 = \frac{30}{366}, p_{10} = \frac{31}{366}, p_{11} = \frac{30}{366}, p_{12} = \frac{31}{366}
\]

\( H_a: \) At least one of the population proportions is not correct as written

Method (M):

Because the answers to the four key questions are (Q) hypothesis testing, (S) sample data, (T) one categorical variable with more than two categories, and (N) one sample, a chi-square goodness-of-fit test will be considered.

The test statistic for this test is

\[
X^2 = \sum_{\text{all categories}} \frac{(\text{observed count} - \text{expected count})^2}{\text{expected count}}
\]

When the null hypothesis is true, this statistic has approximately a chi-square distribution with \( k - 1 \) degrees of freedom.

A significance level of \( \alpha = 0.05 \) will be used for this test.

Check (C):

We are told (in exercise 15.41) to assume these bicycle accidents are a random sample of fatal bicycle accidents. In addition, the table below shows the expected counts in each category, which are all greater than 5, so the sample size is large enough.
The two conditions necessary to use the chi-square goodness-of-fit test have been satisfied.

Calculations (C):

Test statistic:

\[
X^2 = \sum \frac{(\text{observed count} - \text{expected count})^2}{\text{expected count}}
\]

\[
= \frac{(38 - 60.8989)^2}{60.8989} + \frac{(32 - 56.9699)^2}{56.9699} + \cdots + \frac{(42 - 58.9344)^2}{58.9344} + \frac{(40 - 60.8989)^2}{60.8989}
\]

\[
= 8.6103 + 10.9443 + \cdots + 4.8660 + 7.1720
\]

\[
= 78.5105
\]

Degrees of freedom: \( df = k - 1 = 12 - 1 = 11 \)

\( P \)-value: The \( P \)-value is the area under the chi-square curve with 11 degrees of freedom and to the right of 78.5105. Therefore, the \( P \)-value is equal to \( P(X^2 \geq 78.5105) \approx 0 \).

Communicate Results (C):

Because the \( P \)-value of approximately 0 is less than the chosen significance level of \( \alpha = 0.05 \), we reject \( H_0 \). There is convincing evidence that fatal bicycle accidents do not occur in the twelve months in proportion to the lengths of the months.
15.44: Using the five-step process (HMC³):

Hypotheses (H):

We want to determine if there is convincing evidence that the distribution of responses over the distance from home categories is not the same for academic superstars and solid performers.

The hypotheses of interest are:

\[ H_0: \text{ proportions are the same for academic superstars and solid performers } \]
\[ H_a: \text{ proportions are not the same for academic superstars and solid performers } \]

Method (M):

The purpose of this study is to compare five response categories (distance from home) from two populations (academic superstars and solid performers), so a chi-square test of homogeneity will be considered.

The test statistic for this test is

\[ \chi^2 = \sum_{\text{all categories}} \frac{(\text{observed count} - \text{expected count})^2}{\text{expected count}} \]

When the null hypothesis is true, this statistic has approximately a chi-square distribution with \((\text{number of rows} - 1)(\text{number of columns} - 1)\) degrees of freedom.

A significance level of \(\alpha = 0.05\) will be used for this test.

Check (C):

We are told to assume that the students in each sample were randomly selected. The table below shows observed counts and, in parentheses, expected counts. Note that all expected counts are at least 5.

<table>
<thead>
<tr>
<th>Student Group</th>
<th>Distance of College from Home (in miles)</th>
<th>Less than 40</th>
<th>40 to 99</th>
<th>100 to 199</th>
<th>200 to 399</th>
<th>400 or more</th>
</tr>
</thead>
<tbody>
<tr>
<td>Academic Superstars</td>
<td></td>
<td>157</td>
<td>157</td>
<td>141</td>
<td>149</td>
<td>224</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(171.38)</td>
<td>(165.47)</td>
<td>(146.43)</td>
<td>(140.52)</td>
<td>(204.21)</td>
</tr>
<tr>
<td>Solid Performers</td>
<td></td>
<td>104</td>
<td>95</td>
<td>82</td>
<td>65</td>
<td>87</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(89.62)</td>
<td>(86.53)</td>
<td>(76.57)</td>
<td>(73.48)</td>
<td>(106.79)</td>
</tr>
</tbody>
</table>
Because we are assuming that the subjects were randomly selected and all expected counts are at least 5, the chi-square test of homogeneity is appropriate.

Calculations (C):

Test statistic:

\[
X^2 = \sum_{\text{all categories}} \frac{(\text{observed count} - \text{expected count})^2}{\text{expected count}}
\]

\[
= \frac{(157 - 171.38)^2}{171.38} + \frac{(157 - 165.47)^2}{165.47} + \cdots + \frac{(65 - 73.48)^2}{73.48} + \frac{(87 - 106.79)^2}{106.79}
\]

\[
= 1.206 + 0.433 + \cdots + 0.979 + 3.668
\]

\[
= 12.438
\]

Degrees of freedom: \( df = (\text{number of rows} - 1)(\text{number of columns} - 1) = (1)(4) = 4 \)

\( P \)-value: The \( P \)-value is the area under the chi-square curve with 4 degrees of freedom and to the right of 12.438. Therefore, the \( P \)-value is equal to \( P(X^2 \geq 12.438) = 0.014 \).

Communicate Results (C):

Because the \( P \)-value of 0.014 is smaller than the specified significance level of \( \alpha = 0.05 \), we reject \( H_0 \). There is convincing evidence that the distribution of responses over the distance from home categories is not the same for academic superstars and solid performers.

15.45: Using the five-step process (HMC³):

Hypotheses (H):

We want to determine if there is convincing evidence that the spirituality category proportions are not the same for natural and social scientists.

The hypotheses of interest are:

\( H_0 \): The proportions falling into the spirituality categories are the same for natural and social scientists

\( H_a \): The proportions falling into the spirituality categories are not the same for natural and social scientists
Method (M):

The purpose of this study is to compare two populations (natural scientists and social scientists) on the basis of one categorical variable (degree of spirituality), so a chi-square test of homogeneity will be considered.

The test statistic for this test is

$$X^2 = \sum_{\text{all categories}} \frac{(\text{observed count} - \text{expected count})^2}{\text{expected count}}$$

When the null hypothesis is true, this statistic has approximately a chi-square distribution with \((\text{number of rows} - 1)(\text{number of columns} - 1)\) degrees of freedom.

A significance level of \(\alpha = 0.01\) will be used for this test.

Check (C):

We are told that random samples of both natural and social scientists were selected. The table below shows observed counts and, in parentheses, expected counts. Note that all expected counts are at least 5.

<table>
<thead>
<tr>
<th>Degree of Spirituality</th>
<th>Very</th>
<th>Moderate</th>
<th>Slightly</th>
<th>Not at All</th>
</tr>
</thead>
<tbody>
<tr>
<td>Natural Scientists</td>
<td>56 (50.59)</td>
<td>162 (173.92)</td>
<td>198 (199.21)</td>
<td>211 (203.28)</td>
</tr>
<tr>
<td>Social Scientists</td>
<td>56 (61.41)</td>
<td>223 (211.08)</td>
<td>243 (241.79)</td>
<td>239 (246.72)</td>
</tr>
</tbody>
</table>

Because the samples were independent random samples and all expected counts are at least 5, the chi-square test of homogeneity is appropriate.

Calculations (C):

Test statistic:

$$X^2 = \sum_{\text{all categories}} \frac{(\text{observed count} - \text{expected count})^2}{\text{expected count}}$$

$$= \frac{(56 - 50.59)^2}{50.59} + \frac{(162 - 173.92)^2}{173.92} + \cdots + \frac{(243 - 241.79)^2}{241.79} + \frac{(239 - 246.72)^2}{246.72}$$

$$= 0.578 + 0.816 + \cdots + 0.006 + 0.242$$

$$= 3.091$$

Degrees of freedom: \(\text{df} = (\text{number of rows} - 1)(\text{number of columns} - 1) = (1)(3) = 3\)
*P*-value: The *P*-value is the area under the chi-square curve with 3 degrees of freedom and to the right of 3.091. Therefore, the *P*-value is equal to $P(X^2 \geq 3.091) = 0.378$.

Communicate Results (**C**):

Because the *P*-value of 0.378 is greater than the specified significance level of $\alpha = 0.01$, we fail to reject $H_0$. There is not convincing evidence that the spirituality category proportions are not all the same for natural scientists and social scientists.

15.46: Using the five-step process (**HMC**):

**Hypotheses (**H**):**

We want to determine if there is convincing evidence that there is an association between sex and character type for movie characters who smoke.

The hypotheses of interest are:

- $H_0$: There is no association between sex and character type
- $H_a$: There is an association between sex and character type

**Method (**M**):**

The purpose of this study is to compare two categorical variables (sex and character type) from one population (movie characters who smoke), so a chi-square test of association will be considered.

The test statistic for this test is

$$X^2 = \sum_{\text{all categories}} \frac{(\text{observed count} - \text{expected count})^2}{\text{expected count}}$$

When the null hypothesis is true, this statistic has approximately a chi-square distribution with $(\text{number of rows} - 1)(\text{number of columns} - 1)$ degrees of freedom.

A significance level of $\alpha = 0.05$ will be used for this test.

**Check (**C**):**

We are told to assume that this sample is representative of smoking movie characters. The table below shows observed counts and, in parentheses, expected counts. Note that all expected counts are at least 5.
Because the sample was randomly selected and all expected counts are at least 5, the chi-square test of association is appropriate.

Calculations (C):

Test statistic:

\[
X^2 = \sum \frac{(\text{observed count} - \text{expected count})^2}{\text{expected count}}
\]

\[
= \frac{(255 - 262.07)^2}{262.07} + \frac{(106 - 90.95)^2}{90.95} + \cdots + \frac{(12 - 27.05)^2}{27.05} + \frac{(49 - 41.03)^2}{41.03}
\]

\[
= 0.191 + 2.489 + \cdots + 8.370 + 1.550
\]

\[
= 13.702
\]

Degrees of freedom: \(df = (\text{number of rows} - 1)(\text{number of columns} - 1) = (1)(2) = 2\)

\(P\)-value: The \(P\)-value is the area under the chi-square curve with 2 degrees of freedom and to the right of 13.702. Therefore, the \(P\)-value is equal to \(P(X^2 \geq 13.702) = 0.001\).

Communicate Results (C):

Because the \(P\)-value of 0.001 is less than the specified significance level of \(\alpha = 0.05\), we reject \(H_0\). There is convincing evidence that there is an association between sex and character type.

15.47: Using the five-step process (HMC\(^3\)):

Hypotheses (H):

We want to determine if there is convincing evidence that there is an association between whether or not children are overweight 1 year after and the number of sweet drinks consumed.
The hypotheses of interest are:

\( H_0: \) There is no association between whether or not a child is overweight and the number of sweet drinks consumed

\( H_a: \) There is an association between whether or not a child is overweight and the number of sweet drinks consumed

Method (M):

The purpose of this study is to compare two categorical variables (overweight and number of sweet drinks consumed) from one population (children who were underweight or normal weight at age 2), so a chi-square test of association will be considered.

The test statistic for this test is

\[
X^2 = \sum \frac{(\text{observed count} - \text{expected count})^2}{\text{expected count}}
\]

When the null hypothesis is true, this statistic has approximately a chi-square distribution with \((\text{number of rows} - 1)(\text{number of columns} - 1)\) degrees of freedom.

A significance level of \( \alpha = 0.05 \) will be used for this test.

Check (C):

We are told to assume that the sample of children in this study is representative of 2- to 3-year-old children. The table below shows observed counts and, in parentheses, expected counts. Note that all expected counts are at least 5.

<table>
<thead>
<tr>
<th>Number of Sweet Drinks Consumed per Day</th>
<th>Overweight?</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Yes</td>
<td>No</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>22 (28.92)</td>
<td>930 (923.08)</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>73 (65.22)</td>
<td>2,074 (2,081.78)</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>56 (52.77)</td>
<td>1,681 (1,684.23)</td>
<td></td>
</tr>
<tr>
<td>3 or More</td>
<td>102 (106.09)</td>
<td>3,390 (3,385.91)</td>
<td></td>
</tr>
</tbody>
</table>

Because we are assuming that this is a representative sample and all expected counts are at least 5, the chi-square test of association is appropriate.
Calculations (C):

Test statistic:

\[ X^2 = \sum_{\text{all categories}} \frac{(\text{observed count} - \text{expected count})^2}{\text{expected count}} \]

\[ = \frac{(22 - 28.92)^2}{28.92} + \frac{(930 - 923.08)^2}{923.08} + \cdots + \frac{(102 - 106.09)^2}{106.09} + \frac{(3390 - 3385.91)^2}{3385.91} \]

\[ = 1.656 + 0.052 + \cdots + 0.157 + 0.005 \]

\[ = 3.030 \]

Degrees of freedom: \( df = (\text{number of rows} - 1)(\text{number of columns} - 1) = (3)(1) = 3 \)

\( P \)-value: The \( P \)-value is the area under the chi-square curve with 3 degrees of freedom and to the right of 3.030. Therefore, the \( P \)-value is equal to \( P(X^2 \geq 3.030) = 0.387 \).

Communicate Results (C):

Because the \( P \)-value of 0.387 is greater than the specified significance level of \( \alpha = 0.05 \), we fail to reject \( H_0 \). There is not convincing evidence that there is an association between whether or not a child is overweight and the number of sweet drinks consumed.

15.48: Using the five-step process (HMC^3):

Hypotheses (H):

We want to determine if it is reasonable to conclude that there is an association between gender and the number of vacation days taken for working adults in Canada.

The hypotheses of interest are:

\( H_0: \) There is no association between gender and number of vacation days taken

\( H_a: \) There is an association between gender and number of vacation days taken

Method (M):

The purpose of this study is to compare two categorical variables (gender and number of vacation days taken) from one population (working adults in Canada), so a chi-square test of association will be considered.
The test statistic for this test is

\[ X^2 = \sum \frac{(\text{observed count} - \text{expected count})^2}{\text{expected count}} \]

When the null hypothesis is true, this statistic has approximately a chi-square distribution with \((\text{number of rows} - 1)(\text{number of columns} - 1)\) degrees of freedom.

A significance level of \(\alpha = 0.05\) will be used for this test.

Check (C):

We are told the sample was randomly selected from the population of working adults in Canada. The table below shows observed counts and, in parentheses, expected counts. Note that all expected counts are at least 5.

<table>
<thead>
<tr>
<th>Days of Vacation</th>
<th>Gender</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Male</td>
<td>Female</td>
</tr>
<tr>
<td>None</td>
<td>51 (50.05)</td>
<td>42 (42.95)</td>
</tr>
<tr>
<td>1 – 5</td>
<td>21 (24.76)</td>
<td>25 (21.24)</td>
</tr>
<tr>
<td>6 – 10</td>
<td>67 (78.58)</td>
<td>79 (67.42)</td>
</tr>
<tr>
<td>11 – 15</td>
<td>111 (110.34)</td>
<td>94 (94.66)</td>
</tr>
<tr>
<td>16 – 20</td>
<td>71 (75.89)</td>
<td>70 (65.11)</td>
</tr>
<tr>
<td>21 – 25</td>
<td>82 (75.35)</td>
<td>58 (64.65)</td>
</tr>
<tr>
<td>More than 25</td>
<td>118 (106.03)</td>
<td>79 (90.97)</td>
</tr>
</tbody>
</table>

Because the sample was randomly selected and all expected counts are at least 5, the chi-square test of association is appropriate.

Calculations (C):

Test statistic:

\[ X^2 = \sum \frac{(\text{observed count} - \text{expected count})^2}{\text{expected count}} \]

\[ = \frac{(51 - 50.05)^2}{50.05} + \frac{(42 - 42.95)^2}{42.95} + \cdots + \frac{(118 - 106.03)^2}{106.03} + \frac{(79 - 90.97)^2}{90.97} \]

\[ = 0.018 + 0.021 + \cdots + 1.351 + 1.575 \]

\[ = 9.858 \]

Degrees of freedom: \(df = (\text{number of rows} - 1)(\text{number of columns} - 1) = (6)(1) = 6\)
**P-value:** The $P$-value is the area under the chi-square curve with 6 degrees of freedom and to the right of 9.858. Therefore, the $P$-value is equal to $P(X^2 \geq 9.858) = 0.131$.

Communicate Results (C):

Because the $P$-value of 0.131 is greater than the specified significance level of $\alpha = 0.05$, we fail to reject $H_0$. There is no convincing evidence that there is an association between gender and the number of vacation days taken for working adults in Canada.

It is reasonable to generalize this conclusion to the population of working adults in Canada because that is the population from which the sample was randomly selected.

15.49: (a) $H_0$: The proportions falling in the water consumption categories are the same for males and females versus $H_a$: The proportions falling in the water consumption categories are not the same for males and females. Degrees of freedom are calculated using the formula $(\text{number of rows} – 1)(\text{number of columns} – 1)$. Since there are two rows corresponding to gender (male, female), and five columns corresponding to number of servings of water consumed (none, one, two to three, four to five, six or more), there are $(2 – 1)(5 – 1) = (1)(4) = 4$ degrees of freedom.

(b) The $P$-value for the test was 0.086, which is greater than the new significance level of 0.05. So, for a significance level of 0.05, we do not have convincing evidence of a difference between males and females with regard to water consumption.

15.50: Using the five-step process (HMC$^3$):

**Hypotheses (H):**

We want to determine if there is convincing evidence that the distribution of the consumption of fried potatoes is not the same for males and females.

The hypotheses of interest are:

$H_0$: proportions are the same for males and females

$H_a$: proportions are not the same for males and females

**Method (M):**

The purpose of this study is to compare six response categories (number of times fried potatoes were consumed in the past week) from two populations (males and females), so a chi-square test of homogeneity will be considered.
The test statistic for this test is

\[ X^2 = \sum \frac{(\text{observed count} - \text{expected count})^2}{\text{expected count}} \]

When the null hypothesis is true, this statistic has approximately a chi-square distribution with (number of rows – 1)(number of columns – 1) degrees of freedom.

A significance level of \( \alpha = 0.10 \) will be used for this test.

Check (C):

We must assume that the students in each sample were randomly selected. The table below shows observed counts and, in parentheses, expected counts. Note that all expected counts are at least 5.

<table>
<thead>
<tr>
<th>Number of Times Consumed Fried Potatoes in the Past Week</th>
<th>Male</th>
<th>Female</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>2</td>
<td>15</td>
</tr>
<tr>
<td>1 to 3</td>
<td>(5.87)</td>
<td>(11.13)</td>
</tr>
<tr>
<td>4 to 6</td>
<td>10</td>
<td>15</td>
</tr>
<tr>
<td>7 to 13</td>
<td>(8.63)</td>
<td>(16.37)</td>
</tr>
<tr>
<td>14 to 20</td>
<td>12</td>
<td>10</td>
</tr>
<tr>
<td>21 or more</td>
<td>6</td>
<td>20</td>
</tr>
<tr>
<td></td>
<td>(11.05)</td>
<td>(20.95)</td>
</tr>
<tr>
<td></td>
<td>(8.63)</td>
<td>(19)</td>
</tr>
<tr>
<td></td>
<td>(5.18)</td>
<td>(12)</td>
</tr>
<tr>
<td></td>
<td>(8.63)</td>
<td>(16.37)</td>
</tr>
<tr>
<td></td>
<td>(9.82)</td>
<td></td>
</tr>
</tbody>
</table>

Because we are assuming that the subjects were randomly selected and all expected counts are at least 5, the chi-square test of homogeneity is appropriate.

Calculations (C):

Test statistic:

\[ X^2 = \sum \frac{(\text{observed count} - \text{expected count})^2}{\text{expected count}} \]

\[ = \frac{(2 - 5.87)^2}{5.87} + \frac{(10 - 8.63)^2}{8.63} + \cdots + \frac{(19 - 16.37)^2}{16.37} + \frac{(12 - 9.82)^2}{9.82} \]

\[ = 2.552 + 0.216 + \cdots + 0.424 + 0.484 \]

\[ = 14.153 \]

Degrees of freedom: \( \text{df} = (\text{number of rows} - 1)(\text{number of columns} - 1) = (1)(5) = 5 \)

\( P\)-value: The \( P\)-value is the area under the chi-square curve with 5 degrees of freedom and to the right of 14.153. Therefore, the \( P\)-value is equal to \( P(X^2 \geq 14.153) = 0.015 \).
Communicate Results (C):

Because the \( P \)-value of 0.015 is smaller than the specified significance level of \( \alpha = 0.10 \), we reject \( H_0 \). There is convincing evidence that the distribution of the consumption of fried potatoes is not the same for males and females.

Yes, I agree with the authors’ conclusion that there is a significant difference in consumption of fried potatoes for males and females because the null hypothesis was rejected.

15.51: (a)

<table>
<thead>
<tr>
<th></th>
<th>Napped</th>
<th>Did Not Nap</th>
<th>Row Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Men</td>
<td>283</td>
<td>461</td>
<td>744</td>
</tr>
<tr>
<td>Women</td>
<td>231</td>
<td>513</td>
<td>744</td>
</tr>
</tbody>
</table>

(b) Using the five-step process (HMC³):

Hypotheses (H):

We want to determine if there is convincing evidence that there is an association between gender and napping.

The hypotheses of interest are:

\( H_0: \) There is no association between gender and napping

\( H_a: \) There is an association between gender and napping

Method (M):

The purpose of this study is to compare two categorical variables (gender and napping) from one population (adult Americans), so a chi-square test of association will be considered.

The test statistic for this test is

\[
X^2 = \sum_{\text{all categories}} \frac{(\text{observed count} - \text{expected count})^2}{\text{expected count}}
\]

When the null hypothesis is true, this statistic has approximately a chi-square distribution with (number of rows – 1)(number of columns – 1) degrees of freedom.

A significance level of \( \alpha = 0.05 \) will be used for this test.
Check (C):

We are told to assume that the sample is representative of adult American. The table below shows observed counts and, in parentheses, expected counts. Note that all expected counts are at least 5.

<table>
<thead>
<tr>
<th></th>
<th>Napped</th>
<th>Did Not Nap</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Men</strong></td>
<td>283 (257)</td>
<td>461 (487)</td>
</tr>
<tr>
<td><strong>Women</strong></td>
<td>231 (257)</td>
<td>513 (487)</td>
</tr>
</tbody>
</table>

Because the sample is representative and all expected counts are at least 5, the chi-square test of association is appropriate.

Calculations (C):

Test statistic:

$X^2 = \sum_{\text{all categories}} \frac{(\text{observed count} - \text{expected count})^2}{\text{expected count}}$

$= \frac{(283 - 257)^2}{257} + \frac{(461 - 487)^2}{487} + \frac{(231 - 257)^2}{257} + \frac{(513 - 487)^2}{487}$

$= 2.630 + 1.388 + 2.630 + 1.388$

$= 8.037$

Degrees of freedom: $df = (\text{number of rows} - 1)(\text{number of columns} - 1) = (1)(1) = 1$

$P$-value: The $P$-value is the area under the chi-square curve with 1 degree of freedom and to the right of 8.037. Therefore, the $P$-value is equal to $P(X^2 \geq 8.037) = 0.005$.

Communicate Results (C):

Because the $P$-value of 0.005 is less than the selected significance level of $\alpha = 0.05$, we reject $H_0$. There is convincing evidence that there is an association between gender and napping.

(c) Yes. First, recall that the sample is representative of all adult Americans, so the conclusions drawn from this study can be generalized to the population of all adult Americans. Second, the results of this study provide evidence of an association between gender and napping in the population. The number who napped was greater than expected for men and less than expected for women.